TRUCK DISPATCHING AND FIXED DRIVER REST LOCATIONS

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To my loving wife, Sevigne.

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SUMMARY

This thesis sets out to analyze how restricting rest (sleep) locations for long-haul truckers may impact operational productivity, given hours-of-service regulations. Productivity in this thesis is measured by the minimum number of unique drivers required to feasibly execute a set of load requests over a known planning horizon. When drivers may stop for rest at any location, they may maximize utilization under regulated driving hours. When drivers may only rest at certain discrete locations, their productivity may be diminished since they may no longer be able to fully utilize available service hours. These productivity losses may require trucking firms to operate larger driver fleets.

This thesis addresses two specific challenges presented by this scenario; first, understanding how a given discrete set of rest locations may affect driver fleet size requirements; and second, how to determine optimal discrete locations for a fixed number of rest facilities and the potential negative impact on fleet size of non-optimally located facilities. The minimum fleet size problem for a single origin-destination leg with fixed possible rest locations is formulated as a minimum cost network flow with additional bundling constraints. A mixed integer program is developed for solving the single-leg rest facility location problem. Tractable adaptations of the basic models to handle problems with multiple lanes are also presented.

This thesis demonstrates that for typical long-haul lane lengths the effects of restricting rest to a relatively few fixed rest locations has minimal impact on fleet size. For an 18-hour lane with two rest facilities, no increase in fleet size was observed for any test load set instances with exponentially distributed interdeparture times. For test sets with uniformly distributed interdeparture times, additional required fleet sizes ranged from 0 to 11 percent.

The developed framework and results should be useful in the analysis of truck transportation of security-sensitive commodities, such as food products and hazardous materials,

where there may exist strong external pressure to ensure that drivers rest only in secure locations to reduce risks of tampering.

CHAPTER I

INTRODUCTION

1.1 Background

Dispatch of drivers is constrained in practice by many factors, including government regulation as well as union work rules. Hours-of-service regulations in most developed countries (as well as obvious limits on human performance) require that a single driver must rest, usually for at least 8-10 hours and perhaps multiple times, en route from origin to destination on long, direct service lanes. In such cases, drivers typically travel as far as allowed by regulation before resting. Since any given driver may begin a loaded trip with different remaining hours before required rest, such rests could potentially occur at many different locations en route. This study attempts to understand the productivity impact on carriers when driver rest is restricted to a limited number of designated facilities while en route. More specifically, the impact in terms of the number of unique drivers required to deliver a set of loads will be evaluated when rest is restricted to a fixed set of rest facilities, and methods for determining optimal locations for a given number of facilities will be developed.

This research is partially motivated by growing concerns in the United States regarding the security of en route freight moved by motor carriers. Particular concern has focused on food supply chains, which are served predominately by motor carriers. The potential for intentional tampering with the food supply is of considerable concern for both food producers and transporters. While tamper-evident seals can be used to detect unauthorized access, supply chain managers would like to prevent intrusions in the first place, and providing en route security is a top concern. Truck trailers and their contents are clearly most vulnerable when they are parked and drivers are resting. A security survey conducted in 2005 by the American Trucking Research Institute identified the lack of secure parking during driver rest periods to be the most common concern among food carriers (Brewster [3]). A key research question is then: what is the productivity impact on carriers of allowing driver rests only at specified secure facilities?

Current U.S. Department of Transportation (DOT) motor carrier regulations allow a maximum of $\tau_D = 11$ driving hours during a maximum $\tau_{Duty} = 14$ hour duty period before a driver must rest for a minimum of $\tau_R = 10$ hours; existing research has focused on the analysis of the productivity impacts of changes to these regulations, (see, e.g., Ervin and Harris [14]). Alternatively, this thesis considers problems in which a small number of secure rest facilities are located on each lane over which drivers are dispatched, and drivers may only rest at these facilities. It should be clear that an individual driver might not always fully utilize his allowed hours before resting. This reduction in individual driver utilization may ultimately require a trucking firm to use more drivers to complete a given set of load movements. Some initial insights for this problem and some preliminary results are presented in Morris et al. [21].

1.2 Road Map

To understand the impact of discrete rest locations on driver productivity, we focus on the analysis of two important research problems:

- **Minimum driver dispatching problems** - For a given network of terminals and a given set of rest areas, what is the minimum number of drivers needed to deliver a specified set of loads with fixed dispatch times over some planning horizon? The related problem of determining a feasible assignment of drivers to loads that uses this minimum number of drivers will also be considered.

- **Optimal rest facility location problems** - For a given network of terminals and a specified number of rest areas, k, where should the k rest facilities be located to maximize driver productivity? Given a planning horizon and a set of loads with fixed dispatch times, optimal rest facility locations are defined to be those that yield a minimal solution to the resultant minimum driver dispatching problem. Under certain conditions, optimal rest facility locations may be defined to be those that yield a minimal solution to the minimum driver dispatching problem resulting from *any* load set instance.

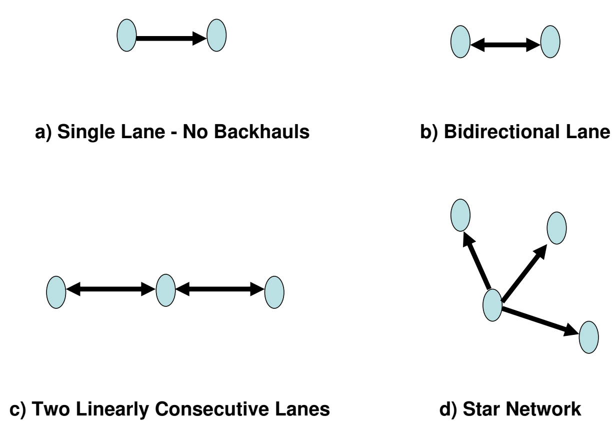

Figure 1: Terminal Network Configurations Analyzed in this Thesis

We focus on these two types of problems for a number of simple network structures to gain initial insights. The specific structures to be analyzed in this thesis are illustrated in Figure 1. To develop the basic principles and terminology, we will first evaluate the driver dispatching problem for a single lane with no backhauls. In this problem, all loads are to be transported from a single origin to a single destination as illustrated in Figure 1 part (a), with drivers returning empty from destination to origin in between consecutive loaded dispatches. We develop a modified network flow formulation for solving this problem which contains additional bundling constraints. In addition to being the most basic network, this single lane with no backhauls also is a practical network to study since trucking companies often dedicate trucks and drivers to single lanes on which few or no backhaul opportunities exist.

We will next examine the optimal rest facility location problems for single lane problems. First, we develop and present a mixed integer programming framework for locating rest facilities optimally on a single lane with no backhauls as in Figure 1 part (a). The principles and insights gained will then be applied to the rest facility location problem for a single lane with backhauls as in Figure 1 part (b). Extending the optimal results to more complex networks is challenging. We will therefore propose practical techniques for this problem, and explore methods for bounding the suboptimality of these solutions. Evaluation of two specific network structure, two linearly consecutive lanes and star networks as illustrated in Figure 1 parts (c) and (d), will conclude the chapter.

Finally, we will analyze minimum driver dispatching problems for a single lane with backhauls, two linearly consecutive lanes and star networks.

1.3 Related Literature

1.3.1 Scheduling and Fleet Sizing

While past research has addressed deterministic driver-to-load dispatching problems (see, e.g., Powell et al. [23] and Erera et al. [13]), no research has addressed the problem of dispatching drivers on long-haul lanes given a restricted set of rest locations. Furthermore, the problem of determining an optimal driver pool size for trucking systems has only been addressed by a few researchers (see, e.g. Erera et al. [12] for an example where the carrier network includes only short-haul lanes).

The body of work focusing on driver assignment for large-scale trucking systems is extensive. Most of this work incorporates hours-of-service regulations on drivers in some fashion, but assumes that the available driver fleet is known. Powell [22] presents a stochastic formulation of the dynamic assignment problem with an application to truckload motor carriers. Yang, Jaillet and Mahmassani [30] address multiple vehicle truckload pickup and delivery problems using an offline mixed integer programming solution method with five different rolling horizon reoptimization strategies. Godfrey and Powell [16] and [17] present an adaptive dynamic programming approach for stochastic fleet management problems with single and multiple period travel times. Kim et al. [20] examine large scale dynamic truckload routing, scheduling and load acceptance with priority demands. Each of these works described above address dynamic problems and intend to improve dispatching operations.

In this thesis, we will focus on problems where the loads to be covered have fixed, known dispatch times and the goal is to determine a set of drivers and assignment decisions that can cover those loads; while such a problem is less relevant for practical operational control, we believe it is useful for tactical analysis. In the routing and scheduling literature, a set of problems with a similar feature are the so-called *vehicle scheduling problems.* Given vehicles based at one or multiple depots, and trips to be executed between points in the surrounding region with specific start and completion times, vehicle scheduling problems focus on determining a set of vehicle tours beginning and ending at a depot that cover all trips and minimize some combination of fixed vehicle costs and variable inter-trip empty transfer costs. Freling, Wagelmans and Paixao [15] analyze vehicle scheduling solution methods for the polynomially-solvable single depot case. Desrosiers et al., editors [10] discuss time constrained routing and scheduling for the same problem.

Dell'Amico et al. [9] present several heuristics for solving the NP-hard multi-depot vehicle scheduling problem where a given set of trips are assigned to vehicles stationed at different depots while minimizing the number of vehicles assigned. Relaxing fixed start and completion times, Currie and Salhi [7] present an exact formulation for a multi-terminal vehicle scheduling problem with backhauling and time windows for trip dispatch, but acknowledge that solving the problem to optimality in an acceptable time is only possible for very small instances; they thus also present an adaptive insertion heuristic for the problem.

Research on vehicle scheduling differs from that considered in this thesis in one very important way. In the driver scheduling problems considered herein, we explicitly model DOT hours-of-service regulations. To do so precisely requires that we know additional state information regarding the driver resource assigned to a load (trip), and that this state information will determine the subsequent trip time (and the end state for the driver assigned).

As mentioned earlier, few researchers consider the problem of sizing the required driver pool. However, drivers are only one type of resource required by trucking systems; other research has considered fleet sizing problems for different resources that do not face the same operational constraints. For example, Beujon and Turnquist [2] present a non-linear model for optimizing vehicle fleet size and allocation in a multi-period environment using a minimum cost network flow formulation with a non-linear objective function. Du and Hall [11] also consider equipment fleet sizing and allocation policies, drawing upon rules derived from inventory theory to balance resources among terminals. Finally, two papers by Vis and colleagues ([28], [29]) address equipment fleet sizing problems at container terminals. In the first paper, a network flow model is used to determine the necessary size of an automated vehicle fleet for transporting containers when each container is available at a known time instant, and in the second paper the ideas are extended to problems where containers are available within known time windows.

1.3.2 Service and Physical Network Design

The literature on facility location problems in transportation networks is also very large, specifically with regards to terminal locations and distribution networks. Existing research uses profit maximization, cost minimization, or some multi-attribute trade-off mechanism in determining optimal locations. We will focus on identifying locations to minimize the size of the driver pool, where driver pool size serves as a proxy for operating cost.

For an example of optimally locating intermodal freight hubs, see Racunica et al. [24]. Syam [25] uses a heuristic solution methodology based on Lagrangian relaxation to solve a capacitated facility location problem and extends this in a later work [26] by introducing several logistical cost components such as holding, ordering, and transportation costs in a multi-commodity, multi-location framework. Syam uses heuristic methodologies based on Lagrangian relaxation and simulated annealing in this later study. Canel and Das [4] use a branch and bound algorithm for solving the uncapacitated, multi-period facility location problem where the objective is to maximize profits. Gunnarsson et al [18] introduces a mixed integer program and heuristic methods for larger combined terminal location and ship routing problems.

In Japan the use of public, multi-company logistics centers has been proposed and Taniguchi et al [27] developed a model for determining the optimal size and location of these public logistics terminals using queuing theory and nonlinear programming techniques.

Another closely related set of problems are tactical *service network design problems*. Crainic ([5] and [6]) analyzes service network design in freight transportation and for long haul freight transportation. Dall Orto et al. [8] presents a static version of the single node dynamic network design problem and proposes two tabu search meta heuristics with a learning mechanism in solving the problem. Armacost et al. [1] uses composite variable formulations for express shipment service network design.

No prior research was found which specifically addressed restricting driver rest to only designated locations and the resultant effects on trip times and driver requirements.

1.4 Contributions

The primary contributions of this thesis are presented below:

- We develop an approach for estimating the impact of restricting driver rest to designated locations on driver productivity;
- Contributions relating to bounds on the number of drivers:
  - We develop a new multi-cycle lower bound on the minimum driver pool size required for single-lane dispatching problems with no backhauls. Computational evidence indicates that this multi-cycle bound is tight in more than 83% of the instances analyzed, and within one driver of the optimal in the remaining instances;
  - We prove that the optimal number of drivers can never be more than 2 times the multi-cycle lower bound for lane lengths longer than half the required rest time;
- Contributions relating to solution methods for minimum driver dispatching problems:
  - We develop a network flow formulation with bundling constraints and demonstrate its practicality for solving problems with limited numbers of load source locations, lanes, and rest facility locations per lane. We perform this demonstration on the following network configurations:
    - Single lane with no backhauls,
    - Bidirectional lane (single lane with backhauls),
    - Star network, and
    - Two linearly consecutive lanes;

- Contributions related to rest facility location methods:
  - We develop a mixed-integer programming approach for locating rest facilities on a single lane to minimize the number of drivers needed to deliver a set of loads which takes into consideration hours of service requirements;
  - We show that allowing unrestricted rest locations for empty trucks does not provide any reduction in the number of drivers required for most lane lengths from 5.5 to 22 hours using up to three rest facilities on a single lane with no backhauls;
  - We prove two theorems regarding when optimum rest facility locations can be determined for a single leg with backhauls independently from the loads to be dispatched;
  - We develop a heuristic algorithm for locating facilities on a single lane with backhauls based on the load set and prove a bound on the worst case performance of this heuristic;
  - We develop heuristic algorithms for distributing a designated number of facilities between lanes and for locating those facilities on the lanes based on the load set for both a star network and for two consecutive lanes. We demonstrate empirically that these heuristics perform very well;
- Using the developed methods, we show that for typical lane lengths and configurations, restricting rest to two or three designated facilities is likely to have minimal impact on driver productivity and hence required driver pool sizes, indicating that such a strategy may deserve further consideration as a mechanism to improve the security of truck transportation systems without compromising efficiency.

CHAPTER II

THE SINGLE LANE MINIMUM DRIVER DISPATCHING PROBLEM WITH FIXED REST AREAS

2.1 Introduction

To evaluate the costs and benefits of using different numbers of rest facilities, a means of determining the minimum required number of drivers with a designated number of facilities is needed. To develop basic principles, we will first consider the driver dispatching problem for the basic single lane with no backhauls. In particular, we will determine the minimum number of drivers required and the associated driver-to-load assignments. In this scenario, all loads are to be transported from a single origin to a single destination, with drivers returning empty from destination to origin between consecutive loaded dispatches.

While the methods developed here for solving the single lane driver dispatching problem are effective for any rest area configuration, we will use those configurations which allow for the minimum number of drivers when we evaluate the benefits of adding additional facilities. We will refer to configurations which allow a minimum number of drivers as *optimal rest facility configurations.* Chapter 3 will address how to find these optimal configurations.

Dispatching problems are typically solved with some pool of drivers available immediately and other drivers in transit or currently resting, and therefore not available until some future time. The absolute minimum number of drivers required will always occur when all drivers are initially fully rested and immediately available. Our formulation of the problem will use this initial condition, but can be easily adapted to incorporate drivers in various states of rest.

Some policies will be allowed in our problem definition which while mathematically advantageous, may not be practical. For example, the problem statement does not require any minimum amount of driving time between rests. A driver could come off rest, drive a half hour and rest again. Such a dispatch scenario would rarely be acceptable by a commercial carrier.

2.2 Minimum Driver-to-Load Assignment Problem Definition

A known set of loads, L, must be delivered from location A to location B. The driving time between locations A and B is δ_{AB} hours in either direction. Drivers can drive a maximum of τ_D hours before they must rest a minimum of τ_R hours. Each load ℓ has a designated dispatch time, τ_ℓ, with the assumption that $\tau_\ell \leq \tau_{\ell+1}$ for all ℓ. Loads must be delivered by $\tau_\ell + \delta_{AB} + (\rho^1_{min} + 1)\tau_R$, where ρ^1_{min} is defined below. Driver rest may only occur at locations A, B, and designated rest areas, R, located $d_r < \delta_{AB}$ hours from location A for $r = 1, 2, ...|R|$.[1] The set of rest area *locations* will be referred to as $d(R)$ and individual rest facilities will be referred to as r_1, r_2, ..., $r_{|R|}$. The *minimum driver-to-load assignment problem* is then to determine the minimum number of drivers required to serve all loads, and the assignment of this set of drivers to loads. Note that we ignore any other rest regulations. For example, we do not consider the "long rest" provisions in current U.S. regulations that restrict the maximum number of driving hours allowed to 60 hours in seven days and 70 hours in eight days.

Definition 2.1. *A* ***lane*** *consists of the origin terminal, A, the destination terminal, B, and hours to drive from one to the other, δ_{AB}.*

Definition 2.2. *A* ***feasible instance*** *of the problem consists of a lane, a load set consisting of the pickup times for each load, and a set of k rest facilities located such that the distance from A to the first rest facility, d_1, the distance between adjacent rest facilities, and the distance from the last rest facility to B are each within the allowable single day driving distance, τ_D.*

In mathematical terms, $d_1 \leq \tau_D$, $d_2 - d_1 \leq \tau_D$,...,$\delta_{AB} - d_k \leq \tau_D$.

Definition 2.3. *The* ***minimum number of one-way rests****, denoted $\rho^1_{min}(R, \delta_{AB}, \tau_D)$, and abbreviated as ρ^1_{min}, is the fewest rests which a fully rested driver will need when dispatched from A to B given rest facility locations $d(R)$.*

[1] In the remainder of this thesis, all distances will be measured in drive hours.

Definition 2.4. *The **maximum number of one-way rests**, denoted* $\rho^1_{max}(R, \delta_{AB}, \tau_D)$*, and abbreviated as* ρ^1_{max}*, is the largest number of rests a driver with any remaining drive hours may need when dispatched from* A *to* B *given rest facility locations* $d(R)$*.*

Definition 2.5. *The **minimum number of rest facilities** required in a feasible instance, denoted* ϕ_{min}*, is given by the following expression:*

$$\phi_{min} = \left\lceil \frac{\delta_{AB}}{\tau_D} \right\rceil - 1 \tag{1}$$

Note that ϕ_{min} rest areas are required since fully-rested drivers can travel only τ_D hours between rests; feasible locations for such facilities are $d_1 = \tau_D$, $d_2 = 2\tau_D$, ..., $d_{\phi_{min}} = \phi_{min}\tau_D$.

2.3 Assumptions

To make the problem tractable without reducing model realism, the following simplifying assumptions are made:

1. Each load may be served only by a driver with sufficient remaining drive hours to reach the first rest area. This eliminates the possibility of a driver returning with no remaining drive hours, picking up a load, then resting before starting the trip to deliver the load. This is not done in practice and therefore is not allowed by our model.

2. When drivers rest during a trip, they will rest only the minimum required time before continuing. Thus, every driver dispatched from A with the same remaining drive hours will always return to A with the same remaining drive hours. This restriction does not reduce driver productivity, since for any optimal solution in which a driver rests longer than the minimum required time, there exists an alternate optimal solution with the same driver-to-load assignments in which the driver rests only the minimum required time.

3. Duty time restrictions are ignored. Ignoring such restrictions may seem like an oversimplification, but under the following conditions, any driver-to-load assignment solution to this relaxed problem will also be a valid driver-to-load assignment (and thus optimal) for the problem with duty restrictions enforced:

   (a) At least one rest is required in each direction on a lane.

   (b) Non-driving duty time at the destination, i.e., truck unloading, is less than or equal to $\tau_{Duty} - \tau_D$, where τ_{Duty} are the maximum duty hours allowed between rests.

   (c) Non-driving duty time to execute a load pick up, i.e., truck loading time, is less than or equal to $\tau_{Duty} - \tau_D$.

   The same is true under the following different conditions:

   (a) At least one rest is required during each round-trip.

   (b) Non-driving duty time at A plus non-driving duty time at B is less than or equal to $\tau_{Duty} - \tau_D$.

   Given a feasible driver-to-load assignment, converting that solution to one which does not exceed duty time restrictions is straightforward when the above conditions hold. Since A is the only location where drivers wait on duty with remaining drive hours, and thus the only location that can result in violating duty time restrictions, the following conversion algorithm solves the problem:

   - For any driver who exceeds duty time limits while waiting at A for a new load, increase that driver's previous rest by the number of non-driving duty hours at A. The driver then returns just in time to pick up the next assigned load.

   This simple algorithm eliminates the non-driving duty time at A at the cost of a longer rest on the road. τ_R is an upper bound on the number of hours that rest would have to be extended because any longer period would have allowed the driver to rest at A before picking up the new load.

2.4 Bounds

Before addressing the minimum driver-to-load assignment problem directly, some simple bounds on the minimum number of drivers required in any feasible solution are developed.

Definition 2.6. *The* ***minimum number of round-trip rests****, denoted* $\rho_{min}(R, \delta_{AB}, \tau_D)$*, is the fewest rests which a fully rested driver must take between successive load assignments at* A *given rest locations* $d(R)$*, lane length* δ_{AB}*, and allowable drive time* τ_D*. For a given instance, this will be referred to as simply* ρ_{min}*.*

Definition 2.7. *The* ***minimum cycle time****, denoted* CT_{min}*, is the minimum time it takes a fully rested driver to complete a round-trip move* $A-B-A$ *and be ready to serve the next load, and is given by the following expression:* $CT_{min} = 2\delta_{AB} + \tau_R * \rho_{min}$*.*

Proposition 2.1. *A lower bound on* ρ_{min} *is given by the following expression:*

$$\rho_{min} \geq \left\lceil \frac{2\delta_{AB}}{\tau_D} - 1 \right\rceil . \tag{2}$$

Proof. Let $R' \subseteq R$ be a set of rest areas used by a driver to transit the distance $2\delta_{AB}$ using the minimum number of rests and let $r' = 1, 2, ..., k$ index this set. Define $d'_{r'}$ as the driving hours from the origin to rest r'. Define $d'_0 = 0$ and $d'_{k+1} = 2\delta_{AB}$. Then the following must be true:

$$d'_i - d'_{i-1} \leq \tau_D \;\; for \;\; i = 1, 2, 3, ..., k+1 \tag{3}$$

Now

$$(d'_1) + (d'_2 - d'_1) + ... + (d'_k - d'_{k-1}) + (d'_{k+1} - d'_k) = d'_{k+1} = 2\delta_{AB} \tag{4}$$

The $k + 1$ terms on the left hand side are each less than or equal to τ_D, so:

$$(k+1)\tau_D \geq 2\delta_{AB} \tag{5}$$

$$k \geq \frac{2\delta_{AB}}{\tau_D} - 1 \tag{6}$$

Since k is an integer,

$$k \geq \left\lceil \frac{2\delta_{AB}}{\tau_D} - 1 \right\rceil \tag{7}$$

□

Clearly, this bound is tight when drivers are allowed to rest anywhere.

Given a set of rest locations $d(R)$, ρ_{min} can be determined exactly by a single-pass forward simulation of a driver who starts fully rested with τ_D remaining drive hours at A as follows: the first rest location will be the furthest reachable rest location, the second rest location will be the furthest reachable rest location after resting at the first, etc. Any driver with less than τ_D remaining drive hours at A would never be able to reach a further point after the same number of rests than the fully-rested driver, and therefore cannot rest fewer times.

Definition 2.8. *The* ***maximum number of round-trip rests****, denoted ρ_{max}, is the most rests a driver may be required to take between successive load assignments given the set of rest locations $d(R)$.*

Definition 2.9. *The* ***maximum cycle time****, denoted CT_{max}, is the maximum required time for any driver to complete a round-trip move $A - B - A$ and be ready to serve the next load, and is given by the following expression: $CT_{max} = 2\delta_{AB} + \tau_R * \rho_{max}$.*

Proposition 2.2. *An upper bound on ρ_{max} is given by: $\rho_{max} \leq \rho_{min} + 1$.*

Proof. Let r be the first rest area used by a fully rested driver at A who drives as far as possible before resting. Clearly, $d_r \leq \tau_D$. Any other driver resting first at r will use the same rest areas as the fully rested driver, and will thus use the same number of rests, ρ_{min}. Alternatively, any driver who must rest once before reaching r could reach r on his second driving day, and then rest. From this point on, the ρ_{min} rest locations would be the same as those of the fully rested driver. □

Given a set of rest locations $d(R)$, ρ_{max} can be determined by continuing the same forward simulation used to determine ρ_{min}. That simulated driver on his first trip will return with some number of remaining drive hours, x. If x hours are not sufficient to reach

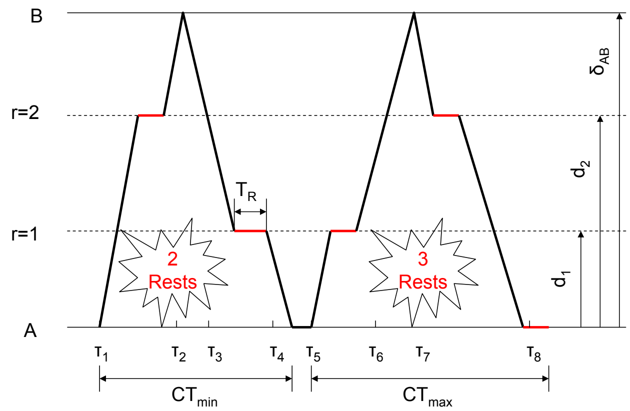

Figure 2: Lane Length, Rest Locations and Cycle Times Illustration

the first rest area, then all drivers must rest at A between trips, and $\rho_{min} = \rho_{max}$ which includes a rest at A. Otherwise, continue to forward simulate the next trip starting with x remaining drive hours, and repeat this trip simulation until either $\rho_{min}+1$ rests are needed, or the number of remaining drive hours at A is the same as in any earlier trip. As an example, let $\delta_{AB} = 18$ hours. Suppose three rest facilities are located 3, 11, and 14 hours from A. A fully rested driver on his first load would stop at rest area 11 on the delivery leg, and at 14 and 3 on the return leg. Clearly, $\rho_{min} = 3$ and his cycle time would be 66 hours, 36 hours driving and 30 hours resting. Since he returns with 8 hours remaining, on his next load he will stop at rest area 3 and 14 on the delivery leg and at 11 on the return leg. Since he returns to A with no remaining drive hours, he must rest again before proceeding. This trip thus requires $\rho_{max} = 4$ rests, and the cycle time is 76 hours: 36 hours driving and 40 hours resting. The reader is referred to Figure 2 for an illustration of the problem parameters and definitions. Note that the first cycle time required after the driver is assigned to load 1 at τ_1 is CT_{min}, while the next cycle time required after assignment to load 5 at τ_5 is CT_{max}.

Definition 2.10. *The* ***minimum number of drivers*** *needed to deliver a set of loads for a given set of rest facility locations $d(R)$ is denoted z^*.*

Any driver assigned to a load will not be available to start a second load until at least CT_{min} hours later. This fact motivates a simple lower bound based on the minimum cycle time.

Proposition 2.3. *A single cycle lower bound on z^*, denoted Λ_1, is given by the following expression:*

$$z^* \geq max_{\ell \in L} \left(\sum_{i \in L} I(\tau_\ell \leq \tau_i < \tau_\ell + CT_{min}) \right) = \Lambda_1, \tag{8}$$

where $I(x)$ is the indicator function: $I(x) = 1$ if x is true and 0 otherwise.

Proof. Assume all loads can be delivered with k drivers and that $k < \Lambda_1$. Furthermore, there exists some time interval that contains Λ_1 loads to be dispatched. Clearly, none of the k drivers assigned to the first k loads in this interval can serve the Λ_1-th load, contradicting our assumption. □

Using similar logic, the maximum possible cycle time defines an upper bound on z^*.

Proposition 2.4. *A cycle upper bound on z^*, denoted Ω_1, is given by the following expression:*

$$z^* \leq max_{\ell \in L} \left(\sum_{i \in L} I(\tau_\ell \leq \tau_i < \tau_\ell + CT_{max}) \right) = \Omega_1. \tag{9}$$

Proof. Let Ω_1 drivers be assigned to loads according to the following assignment algorithm: Drivers are assigned to loads in order of their index numbers. Driver 1 is assigned to load 1, driver 2 is assigned to load 2, and so on. Once the final driver is assigned to a load, start the sequence again with driver 1 assigned to the next load, and so forth. For this assignment algorithm to produce an infeasible solution, there must exist a driver who has been assigned two loads with dispatch times separated by less than CT_{max} hours. Assume this is the case and that the two loads for which this condition holds are loads j and k. Since loads are assigned to the Ω_1 drivers in sequence, then $k - j = \Omega_1$. However, if loads j, $j+1, , k$ are all in the interval $[\tau_j, \tau_j + CT_{max})$ then the minimum number of drivers $\Omega_1 \geq k - j + 1$, which is a contradiction. □

2.5 Driver States

Determining z^* precisely for a given set of loads is not trivial. It should be clear from the discussion that all drivers ready for dispatch at A may not have the same state, and therefore that any simple dispatch rule may not lead to an optimal z^*. A driver's remaining drive hours at the dispatch time of a new load will determine the furthest rest area the driver can reach, the number of rests the driver will need during the round trip, his remaining drive hours after finishing the trip, and the trip's cycle time. When a driver returns to A with sufficient hours to drive onward to a rest facility, there are two possibilities: (1) the driver can be assigned to a load before τ_R hours have elapsed, or (2) the driver can rest at A and once again be a fully rested driver after τ_R hours. Alternatively, a driver may be forced to rest at A if he does not have sufficient hours remaining to reach the first rest area. Thus, one determinant of driver state is remaining drive hours. In fact, the key state variable is the furthest rest area reachable given these remaining hours. For example, if two drivers have 7 and 8 remaining drive hours respectively, and the rest areas are at 3, 4 and 10 hours from A, then both drivers con reach the rest facility at 4 hours from A and thus the drivers have the same state. Interestingly, if drivers drive as far as possible each driving period, they will move through a fixed sequence of (not necessarily unique) states as long as they are dispatched without rest at A; as soon as rest at A is taken (either by necessity or choice), the driver once again becomes fully rested and the sequence is reset. This sequence of furthest-rest-area states is a function of both R and τ_D, and will take one of two forms. If the final unique state in the sequence leads to mandatory rest at A (since the nearest rest area is not reachable), then the sequence will repeat from the fully-rested initial state. Alternatively, a driver dispatched in some state may return to A with a remaining drive hour state that has already been visited; in this case, the sequence will cycle indefinitely without repeating one or more of the initial states. Thus, each driver k available at A at a given time has two distinguishing state parameters, f_k and h_k, where f_k is the furthest reachable rest area, and h_k is the number of hours until he is fully rested. The following proposition follows by definition.

Proposition 2.5. *The maximum number of unique driver furthest-rest-area states is less than or equal to the number of rest facilities, including A and B, within τ_D hours of A. For the purposes of this proposition, rest areas reachable on the return leg are treated as separate rest areas and counted again.*

Definition 2.11. *$\mathcal{F}(R, \mathcal{L})$ is the set of all possible driver furthest-rest-area states for a given rest area configuration R on lane $\mathcal{L}$. This set will often be expressed as simply $\mathcal{F}$ when the lane and rest area configuration are clearly understood.*

Obviously, the maximum number of unique driver furthest-rest-area states can never exceed $2k + 2$, where k is the number of rest facilities. As an example, suppose $\tau_D = 11$, $\delta_{AB} = 4$ and $d_1 = 2$. Then, there are at most four such states. The four rest facilities within τ_D drive hours of A are r_1, B, r_1, and A.

The furthest-rest-area state sequence for a given lane can be determined by simple forward simulation where drivers drive as far as possible between rests. Beginning with a fully-rested driver (in state 0), the last rest facility at which he rests will determine his remaining drive hours and thus his furthest-rest-area state upon return to A, denoted state 1. Then, a state 1 driver is simulated forward. This process continues until one of the previously visited states is revisited, at which point the full sequence is determined. This sequence of furthest reachable rest area states defines a *cycle time sequence*.

We now illustrate these ideas with an example. Consider a lane where δ_{AB} is 18 hours, and where 4 rest areas are located at 3, 8, 11, and 14 hours from A, as illustrated in Figure 3. A fully rested driver delivering a load would rest at 11 on the delivery leg and at 14 and 3 on the return leg. This driver would have 8 hours of drive time remaining and could immediately start another trip. The cycle time on this first trip is 66 hours. If another load is not assigned in the next 10 hours after returning, the driver would rest and revert back to the fully rested state. If the driver is assigned to another load within 10 hours of returning, he would rest at 8, then at B, and then again at 8 on the return leg and have 3 remaining drive hours. This cycle time is also 66 hours. If assigned another load the driver rests at 3 and 14 on the delivery leg and at 11 on the return leg and having no remaining

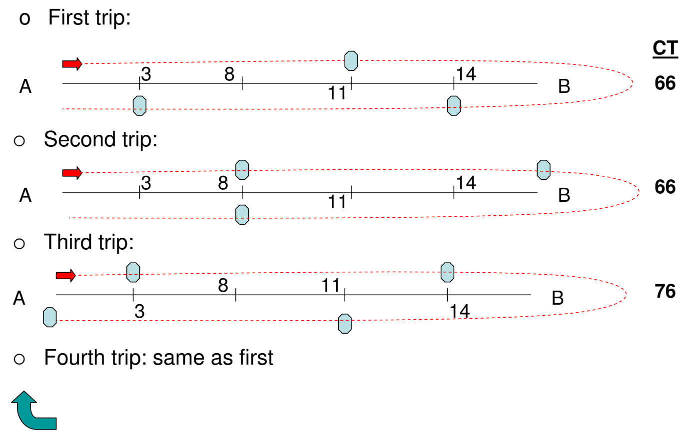

Figure 3: Driver State Sequence Example

Table 1: Driver Furthest-Rest-Area States for an 18-Hour Lane with Rest Facilities at 3, 8, 11, and 14 Hours From Terminal A

State	Cycle Time (hours)	Subsequent State
0	66	1
1	66	2
2	76	0

drive hours, must rest at A for a cycle time of 76 hours. A summary of this driver state sequence is presented in Table 1.

Figure 4 provides an illustration of the structure of all possible driver state sequences. The first state in any state sequence is the fully rested state. All state sequences consist of two sets of states. The first set, which may be empty, contains the initial non-repeating states. The second set contains those states that repeat.

2.6 A Multi-Cycle Bound

A potentially tighter lower bound on z^* can be obtained by looking at a larger window of loads. Specifically, starting with the fully rested state, add the cycle times of the first m driver states. Using this sum as the length of the time window, find the largest number of loads in the load set L within such a window. The number of loads divided by m, rounded

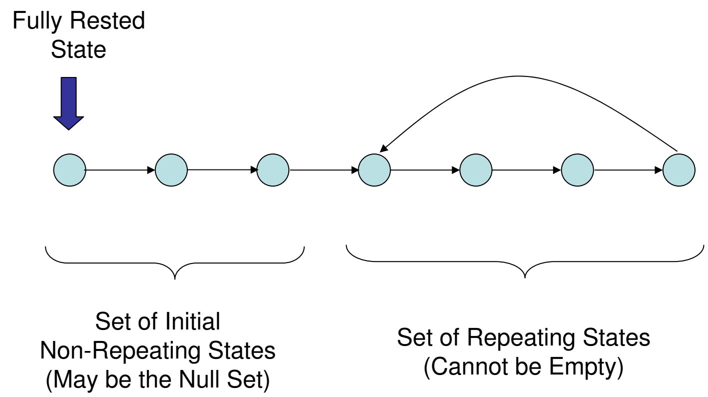

Figure 4: Driver State Sequence Basic Model

up for any remainder, is a lower bound on the number of drivers needed.

Proposition 2.6. *Let CT_i be the cycle time for the i^{th} driver state in a driver state sequence. An m-cycle lower bound on z^*, denoted Λ_m, is given by the following expression:*

$$\Lambda_m = \left\lceil \frac{MAX_{\ell \in L} \sum_{i \in L} I(\tau_\ell \leq \tau_i < \tau_\ell + \sum_{c=1}^{m} CT_c)}{m} \right\rceil \leq z* \tag{10}$$

Proof. The proof of this bound is straightforward since no driver can be assigned more than m loads in any m-cycle period. If each of k drivers can be assigned a maximum of m loads in this period, then at least one more driver is required to deliver any loads in excess of $m \cdot k$ loads in the load set. □

The largest bound among the possible values of m is the tightest m-cycle lower bound. Clearly, if the first k cycle times include the entire planning horizon, $m = k$ is the largest m requiring consideration.

Definition 2.12. *The maximum of the m-cycle bounds for all possible values of m is called the* ***multi-cycle lower bound****, and is denoted Λ_M.*

This multi-cycle bound is not guaranteed to be a tight bound as illustrated by the following example. Consider a lane whose cycle time sequence is a series of three cycle times which repeat indefinitely: 14,14,24,14,14,24... Now consider a load set consisting of five loads whose pickup times are: 1, 16, 33, 54, and 69.

- To determine Λ_1, scan this load set for the highest number of loads to be picked up in any window of 14 hours, which is 1.
- To determine Λ_2, scan this load set for the highest number of loads to be picked up in any window of $14+14=28$ hours, which is 2. Divide this by $m=2$ and the bound is 1.
- To determine Λ_3, scan this load set for the highest number of loads to be picked up in any window of $14+14+24=52$ hours, which is 3. Divide this by $m=3$ and the bound is 1.
- To determine Λ_4, scan this load set for the highest number of loads to be picked up in any window of $14+14+24+14=66$ hours, which is 4. Divide this by $m=4$ and the bound is 1.

In this example, each of the m-cycle bounds results in a minimum of one driver required, but the driver delivering the loads at times 1, 16, and 33 cannot deliver the load at time 54, since the pickup time of the third load plus the third cycle time in the cycle time sequence is $33+24=57$, which is past the time 54 pickup time for the 4^{th} load. So at least two drivers are required.

2.7 Worst Case Performance of Bounds

The performance of the upper and lower bounds on the number of drivers depends on the lane length as well as the required rest time, τ_R, as described in the following theorem.

Theorem 2.1. *On a single lane with all loads originating at A, the ratio of the single cycle upper bound to the single cycle lower bound on the number of drivers required is bounded by the following expression:*

$$\frac{\Omega_1}{\Lambda_1} \leq \left\lceil \frac{\tau_R}{2\delta_{AB}} \right\rceil + 1 \tag{11}$$

Proof. By the definitions of the single cycle upper and lower bounds:

$$\frac{\Omega_1}{\Lambda_1} = \frac{\textit{Max Loads in Window of Length } CT_{max}}{\textit{Max Loads in Window of Length } CT_{min}} \tag{12}$$

Since $CT_{max} \leq CT_{min} + \tau_R$:

$$\frac{\Omega_1}{\Lambda_1} \leq \frac{\textit{Max Loads in Window of Length } (CT_{min} + \tau_R)}{\textit{Max Loads in Window of Length } CT_{min}} \tag{13}$$

$$\frac{\Omega_1}{\Lambda_1} \leq 1 + \frac{\textit{Max Loads in Window of Length } \tau_R}{\textit{Max Loads in Window of Length } CT_{min}} \tag{14}$$

Since $CT_{min} \geq 2\delta_{AB}$:

$$\frac{\Omega_1}{\Lambda_1} \leq 1 + \frac{\textit{Max Loads in Window of Length } \tau_R}{\textit{Max Loads in Window of Length } 2\delta_{AB}} \tag{15}$$

The window of length τ_R can be broken up into segments of length CT_{min}, with any remainder making another segment. The number of these segments is at most $\lceil \tau_R/2\delta_{AB} \rceil$. Further, for k segments, k times the maximum number of loads in any single segment is at least as large as the number of loads in all of the segments combined. Therefore:

$$\frac{\Omega_1}{\Lambda_1} \leq 1 + \frac{\lceil \frac{\tau_R}{2\delta_{AB}} \rceil \textit{Max Loads in Window of Length } 2\delta_{AB}}{\textit{Max Loads in Window of Length } 2\delta_{AB}} \tag{16}$$

and

$$\frac{\Omega_1}{\Lambda_1} \leq \left\lceil \frac{\tau_R}{2\delta_{AB}} \right\rceil + 1. \tag{17}$$

□

The following corollaries follow from this result and are presented without proof.

Corollary 2.1.1. $\frac{\Omega_1}{\Lambda_1} \leq 2$ *for lanes longer than* $\tau_R/2$.

Corollary 2.1.2. *For lanes longer than* $\tau_R/2$*, the number of drivers used by any heuristic which adds a new driver only if no previously used driver is available, is at worst 2 times* z^*.

Corollary 2.1.3. *$z^* \leq 2\Lambda_M$ for lanes longer than $\tau_R/2$.*

The following is an example of when this bound on the ratio of the upper to lower bound is tight. Consider an 18-hour lane ($2\delta_{AB} = 36$), with $\tau_D = 11$, $\tau_R = 10$, and three rest facilities which result in a cycle time sequence of 66 hours followed by 76 hours at which point the cycle time sequence begins again. Now consider a load set which requires five loads to be picked up at time 0, five loads to be picked up at time 71 and five loads to be picked up at time 143. The single cycle lower bound on the number of drivers (maximum number of loads in any 66 hour window) is five. The cycle upper bound (maximum number of loads in any 76 hour window) is ten. This is a ratio of two. As a side note, the multi-cycle lower bound is also five and z^* in this example is ten since the five drivers picking up the loads at time 0 and then at 71 will not be available to pick up the loads at time 143. So the ratio of z^* to the multi-cycle lower bound is also two.

2.8 Determining Minimum Driver Solutions

2.8.1 Network Flow Formulation with Bundling Constraints

For a given load set and rest area configuration, finding the minimum number of drivers and an associated driver-to-load assignment appears to be a hard optimization problem. We do not prove formally the complexity of this problem in this dissertation; this remains an open problem. Moderately sized problems can be solved using a tree search algorithm, although this may require very long computing times. We were not able to identify a heuristic with polynomial-time complexity for this problem.

Since solution methods for acyclic network flow models are typically very fast, if this problem could be modeled as a network flow problem, solutions may be achievable in a much shorter time. It turns out that this problem can be modeled as a minimum cost network flow model, but with additional bundling constraints to ensure each load is delivered. Computational evidence suggests that these bundling constraints do not limit the practicality of this solution approach. A discussion of the formulation follows.

2.8.1.1 The Network

Consider a network graph, G, with a single source node, s, and a single sink node, t. Let each load $\ell \in L$ be represented by a set of nodes, one for each possible driver state. Let node (ℓ, f) be the node representing dispatching load ℓ with a driver in furthest-rest-area state $f \in \mathcal{F}$. Let f_0 be the fully rested state and let f_f^S be the state following state f in the furthest-rest-area state sequence. Let CT_f be the cycle time for a load delivered by a driver in furthest-rest-area state f. Let $\ell^T(\ell, t)$ be the first load with a dispatch time greater than or equal to $t_\ell + t$. Driver dispatch decisions and state transitions will be represented by flow on arcs in the network. A single arc emanates from the source node, s, and connects to the fully-rested state node for load 1: $(s, (1, 1))$. Each load-state node, (ℓ, f), in the network has exactly 3 outbound arcs, one from each of the following types:

1. Unassigned arcs (U-arcs): each U-arc connects node (ℓ, f) to the node representing the same state at the next load, $(\ell + 1, f)$.

2. Assignment arcs (A-arcs): each A-arc connects node (ℓ, f) to the node representing the next state in the driver state sequence at the first available load with a dispatch time after the appropriate cycle time, $(\ell^T(\ell, CT_f), f_f^S)$

3. Assignment-with-rest arcs (AR-arcs): each AR-arc connects node (ℓ, f) to the fully rested state at the first load after the appropriate cycle time plus whatever time is needed for the driver to become fully rested, $(\ell^T(\ell, CT_f), f_0)$ if $f_f^S = f_0$ or $(\ell^T(\ell, CT_f + \tau_R), f_0)$ otherwise.

Whenever such an arc would terminate at a time later than the dispatch time of the last load, the arc terminates instead at the sink node. Each intermediate node has a varying number of incoming arcs that depends on the load distribution in relation to cycle times. Let U be the set of all U-arcs in the network, A be the set of all A-arcs, and AR be the set of all AR-arcs.

This network models assignments of drivers to loads by the flow values on individual network arcs. Flow along U-arcs represents drivers not assigned to the load at the source

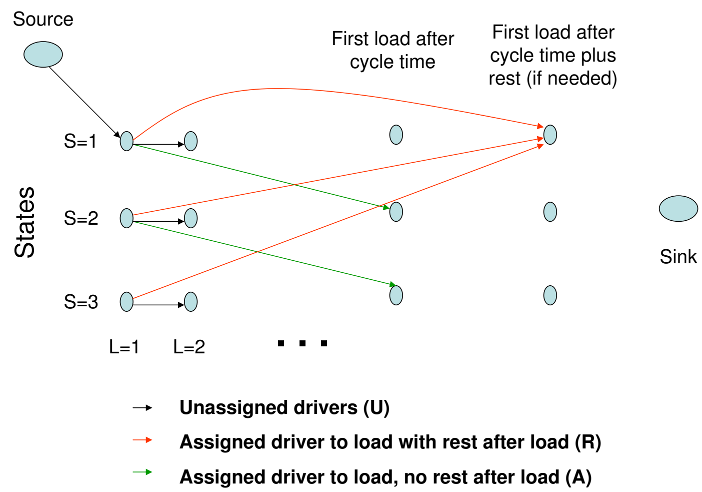

Figure 5: Network Flow Formulation Nodes and Arc Types

of the arc. Flow along an A-arc represents a driver assigned to the load at the beginning of the arc who will not rest before his next assigned load. Flow along an AR-arc represents a driver assigned to the load at the beginning of the arc who will rest after completing the load before being assigned to another load. Figure 5 illustrates these three types of arcs.

This network is a simplification of the actual problem since it does not directly model the fact that drivers revert to the fully rested state if unassigned for a period greater than or equal to τ_R. This network model allows drivers to remain in a higher state for long periods of time by repeatedly following U-arcs in those higher states. However, this does not affect the validity of the model since any load set assigned to a driver starting in a non-fully-rested state could be delivered by a fully rested driver as well. Furthermore, the formulation allows for the rest possibility by use of the AR-arc instead of the A-arc on the previously assigned load.

Consider Figure 6. A driver assigned to load i in state 1 could follow the red AR-arc to load j state 0, or follow the green A-arc to his next feasible load in state 2. If the driver in state 2 remains unassigned for τ_R, the driver should revert to the fully rested state at load j. This is indicated by the dashed arc. The network model, however, keeps the driver in state 2. Suppose then the driver in state 2 is assigned load j in some best solution. Note

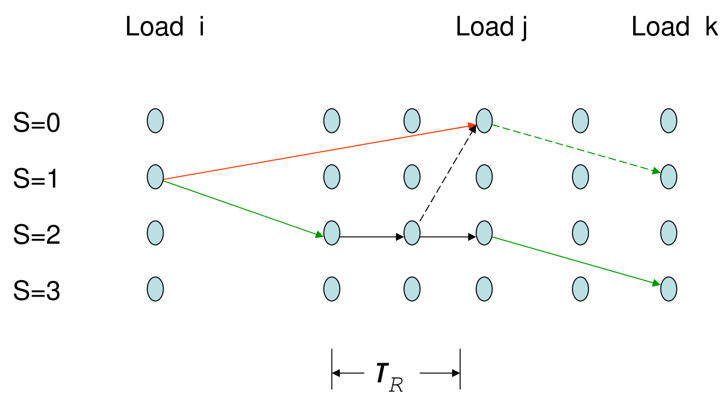

Figure 6: Equivalent Arc Paths in the Network Flow Formulation

that the driver in state 0 could also have been assigned this load, and since the driver in state 0 is fully rested, he will be able to deliver any set of future loads the driver in state 2 could have delivered after completing load j. Of course, our model also allows that the driver could have completed load i using the AR-arc if that would have been advantageous for completing future loads. One advantage of this model is that the state space is much smaller than that which would be required to maintain each individual driver's time to reach the fully rested state, which in the worst scenario could require a different state node for each driver at each load.

Using this network, we now specify a network optimization problem for determining driver-to-load assignments that require the minimum number drivers in the pool.

2.8.1.2 Variables

- y_u^U : Integer variable representing flow on the unassigned arc, $u \in U$.
- y_a^A : Integer variable representing flow on the assignment arc, $a \in A$.
- $y_{a_r}^{AR}$: Integer variable representing flow on the assignment-with-rest arc, $a_r \in AR$.

Each arc, u, a, or a_r, can be written as a directed connection between two nodes, (ℓ, f) and (k, g): $(\ell, f), (k, g)$. For simplicity of notation in the network formulation to follow, let $s \equiv (0, 1)$ and $t = (|L| + 1, 1)$, and define $(0, 1), (1, 1)$ as a U-arc in U.

2.8.1.3 Objective Function

Since the objective is to minimize the number of drivers, we minimize the entry flow from the source node to the network:

$$min\ y^U_{(0,1),(1,1)} \tag{18}$$

2.8.1.4 Constraints

- *Flow Balance Constraints.* For each intermediate node, drivers arriving at the node along any U-arcs, A-arcs, or AR-arcs, must leave along either a U-arc, A-arc, or AR-arc. For each load ℓ and state f:

$$y^U_{(\ell-1,f)(\ell,f)} + \sum_{k=1}^{\ell-1} \sum_{g=1}^{|\mathcal{F}|} (y^A_{(k,g),(\ell,f)} + y^{AR}_{(k,g),(\ell,f)}) = y^U_{(\ell,f)(\ell+1,f)} + \sum_{k=\ell+1}^{|L|+1} \sum_{g=1}^{|\mathcal{F}|} (y^A_{(\ell,f),(k,g)} + y^{AR}_{(\ell,f),(k,g)}) \tag{19}$$

- *Bundling Constraints.* For each load, at least one driver must be assigned. For each load ℓ:

$$\sum_{f=1}^{|\mathcal{F}|} \sum_{k=\ell+1}^{|L|+1} \sum_{g=1}^{|\mathcal{F}|} (y^A_{(\ell,f),(k,g)} + y^{AR}_{(\ell,f),(k,g)}) \geq 1 \tag{20}$$

2.8.1.5 Converting Network Flow Solution to Driver Assignments

The following algorithm easily converts the network flow solution to driver-to-load assignments. This is a standard flow decomposition approach and is included here for completeness:

1. Start with the network graph nodes. Add all arcs with $y^{\{U,A,AR\}} > 0$ in the network flow solution. Set the capacity of each arc equal to the solution flow on that arc.

2. Create a new driver.

3. Starting at the source, follow any path with remaining capacity to the sink.

   (a) As each arc is traversed reduce that arcs capacity by 1.

   (b) If any traversed arc is an A-arc or an AR-arc and that load has not already been assigned to a driver, assign the load to this driver.

4. If the remaining capacity on the arc $(0, 1), (1, 1)$ is 0, stop. Otherwise, repeat from step 2.

2.8.2 Online Heuristics

Since the optimal determination of z^* may be computationally prohibitive for large problems, especially those with a large state space, several simple online driver-to-load assignment heuristics were developed to be implemented as an alternative to the network flow optimization approach. Each heuristic simultaneously determines a number of drivers and a feasible driver-to-load assignment by processing each load exactly once. The online heuristic approach has a few practical advantages over the network flow approach. Dispatching decisions are made by processing loads sequentially without looking forward in time. In most real dispatching problems, new load requests are received continuously and the set of loads to be dispatched over a given time frame is constantly changing. While this poses a problem for the network flow approach since re-optimization is required after each change, this has no effect on online heuristics, since future loads do not play a role in the dispatching decision. For a given load, the heuristic will assign an available driver at A based on a simple rule, and if no driver is available, a new driver is created and assigned. These heuristics are also useful in providing a potentially tighter upper bound on the number of drivers required. The following heuristic rules were considered:

- *Lowest Index* - The load is assigned to the available driver with the lowest index. For example, if drivers 1,7, and 14 are available to deliver a specified load, the load will be assigned to driver 1.

- *Lowest State* - The load is assigned to the driver in the furthest-rest-area state with the lowest index. Lower index states typically have lower cycle times, allowing the driver to return more quickly for future loads while also allowing drivers in higher index states to potentially avoid assignment long enough to return to the fully-rested initial state.
- *Highest State* - The load is assigned to the driver in the highest-index state.
- *Last-In First-Out (LIFO)* - The load is assigned to the available driver who has most recently finished his previous load, again to allow the drivers who have been back longer the potential to return to the fully-rested state.
- *First In First Out (FIFO)* - The load is assigned to the available driver who has been available the longest, attempting to balance the loads between drivers.
- *Lowest State - LIFO Tiebreaker* - The lowest-state heuristic is used, with ties broken using the LIFO condition.
- *Highest State - LIFO Tiebreaker* - The highest-state heuristic is used, with ties broken using the LIFO condition.
- *Lowest Non-rested State* - The load is assigned to the available driver with the lowest-index state other than fully-rested.
- *Lowest Cycle Time* - The load is assigned to the available driver who will be able to deliver the load and be available for a following load in the least amount of time.
- *Lowest Cycle Time - LIFO Tiebreaker* - The load is assigned using the lowest cycle time heuristic, breaking ties using the LIFO rule.
- *Load Density Cycle Time* - This heuristic tries to use the lowest cycle time heuristic with the LIFO tiebreaker when it is most beneficial, namely, when the number of loads in the near future is high. When the number of loads in the near future is low, it assigns a driver with highest cycle time, again with the LIFO tiebreaker.

After applying the rules, all ties are broken by the lowest index rule.

Definition 2.13. *For a given lane length δ_{AB}, set of rest area locations, $d(R)$, load set L, τ_D, and τ_R, the heuristic upper bound on z^*, denoted $\Omega_H(L, \delta_{AB}, d(R), \tau_D, \tau_R)$, or simply Ω_H is the minimum number of drivers used on this lane and this load set by any of the above heuristics.*

2.9 Computational Experiment

Using the developed techniques, an analysis of the productivity impacts of restricting rest locations on a single lane is now presented. The effects of several variables were evaluated: lane length, mean inter-arrival time (IAT) between successive load dispatch times, variability of the IAT's, and the type of distribution for the IAT's. Two typical trucking lane lengths were chosen for the analysis: 9 and 18 hours.

For our computational study, we compare the impact of the number of rest facilities on required driver pool size. For a given number of rest facilities, the facilities were optimally located as will be discussed in the next chapter. The effect of locating the facilities in non-optimal locations will also be evaluated in the next chapter.

2.9.1 Rest Facilities

For an 18-hour lane, it is clear that at least one rest facility is needed. When only one rest facility is allowed, it can only be feasibly located at a position such that all drivers available at A will always have the same cycle time, 76 hours. This is because such a facility must be located within 11 drive hours of both A and B, and thus $[7, 11]$ hours from A. Thus, all drivers must always rest at A upon return, and therefore Ω_1 drivers (from Equation (9)) will be needed to deliver any load set. Note that in such cases where all drivers at A always must have the same furthest-rest-area state, each of the online heuristics will provide an optimal minimum driver solution. Similarly, for the 9-hour lane when there are no rest facilities, all drivers rest at both A and B on each cycle with a resulting 38 hour cycle time on every cycle.

For an 18-hour lane, using 6, 7, 8, 9, or 10 rest facilities results in the same cycle time sequence as using 5 rest facilities, and therefore will result in identical solutions to the five rest facility case. Similarly, using 11, 12, 13, 14, 15, or 16 rest facilities results in the same cycle time sequence. Therefore, we present results for 2, 3, 4, 5, 11, and 17 rest facilities on the 18-hour lane. For a 9-hour lane, using 4, 5, 6 or 7 rest facilities results in the same cycle time sequence as using three rest facilities, and therefore will result in identical solutions to the three rest facility case. Therefore, we present results for 1, 2, 3, and 8 rest facilities on the 9-hour lane.

2.9.2 Load Sets

The minimum number of drivers z^* required to serve all loads on a lane depends on the load set. How loads are clustered in time will also affect the separation between the upper and lower bounds on z^*. Additionally, the times required to determine optimal solutions may vary significantly. For this reason, load sets with different inter-arrival time (IAT) characteristics were used in the analysis. The basic characteristics evaluated were:

- *Uniformly distributed versus exponentially distributed IAT's.* Exponential distributions are frequently used to model IAT's, but the nature of this problem should make load sets with exponentially distributed IAT's fairly easy to solve since the likelihood of consecutive cycle time time periods with high load densities is small. More regular IAT's such as uniformly distributed IAT's should pose a more challenging assignment problem. Further, these types of IAT distributions can be found in many supply chains in which supplies are shipped on a regular schedule.

- *Average IAT.* Load sets with 170 minute and 500 minute average IAT's were evaluated.

- *IAT variability.* For the load sets with exponentially distributed IAT's, the variability is determined directly by the mean IAT. For the load sets with uniformly distributed IAT's, 3 different variability levels were evaluated:

  - No variability. All IAT's are the same, i.e., 170 or 500 minutes.

  - Low Variability. IAT's distributed uniformly on the range from the average IAT minus 20 percent to the average IAT plus 20 percent.

Table 2: Parameters of Load Sets Used in the Single Lane Experiment

Load Sets	IAT Distribution	Mean IAT (minutes)	IAT Range
$LU0$	-	170	170
$LU1 - LU5$	Uniform	170	$136 - 204$
$LU6 - LU10$	Uniform	170	$34 - 306$
$U0$	-	500	500
$U1 - U5$	Uniform	500	$400 - 600$
$U6 - U10$	Uniform	500	$100 - 900$
$LE1 - LE5$	Exponential	170	NA
$E1 - E5$	Exponential	500	NA

– High Variability. IAT's distributed uniformly on the range from the average IAT minus 80 percent to the average IAT plus 80 percent.

Table 2 summarizes the characteristics of the load sets considered. Five load set instances were generated for each combination of the above listed characteristics with the exception of the load sets with fixed IAT's (no variability). In these cases, only one instance for each level of IAT was generated since any additional instances would be identical. Each load set contains 500 loads.

2.9.3 Results

Tables 3 through 12 present results for each combination of lane length and number of rest facilities. Each table shows the following statistics for each load set: the single cycle lower bound on the number of drivers required, the multi cycle lower bound, the optimal solution from the network flow model, the upper bound obtained by the heuristic which used the fewest drivers, the single cycle upper bound, and the run time (in seconds) of the mixed integer programming formulation presented in Section 2.8.1 using AMPL/CPLEX on a Sun V480 with 4x900MHz UltraSparc-III-Cu and 16GB of RAM. The run times for the heuristics and the bounds are not included in the tables because they were the same for every scenario. For a given scenario, all heuristics and all bound calculations could be performed in under 1 second on a standard laptop computer with a 1.8 GHz Intel Centrino Duo Core processor.

Although results for a single rest area on the 18-hour lane and no rest areas on the 9-hour lane are not included due to the simplicity of the problem, the number of drivers needed in these cases is the same as the cycle upper bound column in any of the results tables for that lane length.

2.9.3.1 18-Hour Lane, 2 Rest Facilities

Table 3 provides the results for the 18-hour lane with 2 rest facilities. The following are the key observations.

- 25 of the 32 loads sets were solved optimally in 5 seconds or less, and each load set could be solved optimally in under 2 minutes.
- The uniformly distributed load sets with no variability, $U0$ and $LU0$, took the longest to solve by far, requiring 104 seconds each. The next longest run time was 43 seconds.
- Among the load sets with uniformly distributed IAT's, more variability in the IAT's resulted in shorter run times but more required drivers.
- In each of the load sets with exponentially distributed IAT's, the optimal solution was equal to the lower bound.
- The multi-cycle lower bound was tight for all but 4 of the load sets: $LU2$, $LU4$, $LU5$, and $LU9$. For each of these 4 load sets, the multi-cycle lower bound differed from the optimal solution by 1 driver.
- The heuristics provided an optimal solution for all but 8 of the load sets: $LU1$, $LU3$, $LU4$, $LU6$, $LU8$, $LU9$, $LU10$, and $LE2$.
- The largest absolute difference between the heuristic upper bound and the optimal solution was 3 drivers.
- The largest relative difference between the heuristic upper bound and the optimal solution was 10 percent.
- The ratio for the cycle UB to the cycle LB ranged from 1.05 to 1.25.

Table 3: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 18-Hour Lane with 2 Rest Facilities

`Load Set`	Λ_1	Λ_M	z^*	Ω_H	Ω_1	`Run Time (sec)`
$U0$	8	10	10	10	10	104
$U1$	9	10	10	10	11	5
$U2$	9	10	10	10	11	5
$U3$	10	10	10	10	11	3
$U4$	9	10	10	10	10	4
$U5$	9	10	10	10	11	4
$U6$	13	13	13	13	15	1
$U7$	14	14	14	14	15	1
$U8$	12	12	12	12	14	1
$U9$	13	13	13	13	15	1
$U10$	12	12	12	12	14	1
$LU0$	24	27	27	27	27	104
$LU1$	26	28	28	30	30	43
$LU2$	25	27	28	28	29	19
$LU3$	25	27	27	28	29	36
$LU4$	25	27	28	29	29	23
$LU5$	25	27	28	28	29	31
$LU6$	30	30	30	31	34	2
$LU7$	29	29	29	29	33	2
$LU8$	30	30	30	33	33	2
$LU9$	28	28	29	30	33	3
$LU10$	30	31	31	32	34	1
$E1$	20	20	20	20	21	1
$E2$	15	15	15	15	16	1
$E3$	19	19	19	19	21	1
$E4$	20	20	20	20	21	1
$E5$	18	18	18	18	19	1
$LE1$	40	40	40	40	44	1
$LE2$	35	35	35	36	39	1
$LE3$	39	39	39	39	42	1
$LE4$	37	37	37	37	39	1
$LE5$	38	38	38	38	41	1

2.9.3.2 18-Hour Lane, 3 Rest Facilities

Table 4 provides the results for the 18-hour lane with 3 rest facilities. The following are the key observations.

- All load sets were solved optimally in under 5 seconds.
- Among the load sets with uniformly distributed IAT's, more variability in the IAT's resulted in shorter run times but more required drivers.
- In each of the load sets with exponentially distributed IAT's, the optimal solution was equal to the lower bound.
- The multi-cycle lower bound was tight for all but 4 of the load sets: $U0$, $U4$, $LU3$, and $LU5$. For each of these 4 load sets, the multi-cycle lower bound differed from the optimal solution by 1 driver.
- The heuristics provided an optimal solution for all but 6 of the load sets: $LU0$, $LU1$, $LU4$, $LU5$, $LU9$, $LU10$.
- The largest absolute difference between the heuristic upper bound and the optimal solution was 3 drivers.
- The largest relative difference between the heuristic upper bound and the optimal solution was 11 percent.
- The ratio for the cycle UB to the cycle LB ranged from 1.05 to 1.25.

2.9.3.3 18-Hour Lane, 4 Rest Facilities

Table 5 provides the results for the 18-hour lane with 4 rest facilities. The following are the key observations.

- All load sets were solved optimally in under 35 seconds.
- Among the load sets with uniformly distributed IAT's, more variability in the IAT's resulted in shorter run times but more required drivers.

Table 4: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 18-Hour Lane with 3 Rest Facilities

Load Set	Λ_1	Λ_M	z^*	Ω_H	Ω_1	Run Time (sec)
$U0$	8	9	10	10	10	2
$U1$	9	10	10	10	11	2
$U2$	9	10	10	10	11	2
$U3$	10	10	10	10	11	1
$U4$	9	9	10	10	10	2
$U5$	9	10	10	10	11	2
$U6$	13	13	13	13	15	1
$U7$	14	14	14	14	15	1
$U8$	12	12	12	12	14	1
$U9$	13	13	13	13	15	1
$U10$	12	12	12	12	14	1
$LU0$	24	26	26	27	27	4
$LU1$	26	27	27	30	30	1
$LU2$	25	27	27	27	29	1
$LU3$	25	26	27	27	29	1
$LU4$	25	27	27	28	29	2
$LU5$	25	26	27	28	29	2
$LU6$	30	30	30	30	34	1
$LU7$	29	29	29	29	33	1
$LU8$	30	30	30	30	33	1
$LU9$	28	28	28	29	33	1
$LU10$	30	31	31	32	34	1
$E1$	20	20	20	20	21	1
$E2$	15	15	15	15	16	1
$E3$	19	19	19	19	21	1
$E4$	20	20	20	20	21	1
$E5$	18	18	18	18	19	1
$LE1$	40	40	40	40	44	1
$LE2$	35	35	35	35	39	1
$LE3$	39	39	39	39	42	1
$LE4$	37	37	37	37	39	1
$LE5$	38	38	38	38	41	1

Table 5: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 18-Hour Lane with 4 Rest Facilities

Load Set	Λ_1	Λ_M	z^*	Ω_H	Ω_1	Run Time (sec)
$U0$	8	9	9	10	10	33
$U1$	9	9	10	10	11	4
$U2$	9	9	10	10	11	6
$U3$	10	10	10	10	11	2
$U4$	9	9	10	10	10	6
$U5$	9	9	10	10	11	4
$U6$	13	13	13	13	15	1
$U7$	14	14	14	14	15	2
$U8$	12	12	12	12	14	2
$U9$	13	13	13	13	15	2
$U10$	12	12	12	12	14	8
$LU0$	24	25	26	27	27	33
$LU1$	26	26	26	29	30	13
$LU2$	25	26	27	27	29	10
$LU3$	25	25	26	27	29	13
$LU4$	25	26	26	28	29	16
$LU5$	25	26	26	28	29	11
$LU6$	30	30	30	30	34	1
$LU7$	29	29	29	29	33	1
$LU8$	30	30	30	31	33	3
$LU9$	28	28	28	28	33	2
$LU10$	30	30	30	31	34	1
$E1$	20	20	20	20	21	1
$E2$	15	15	15	15	16	1
$E3$	19	19	19	19	21	5
$E4$	20	20	20	20	21	2
$E5$	18	18	18	18	19	1
$LE1$	40	40	40	40	44	1
$LE2$	35	35	35	35	39	2
$LE3$	39	39	39	39	42	1
$LE4$	37	37	37	37	39	2
$LE5$	38	38	38	38	41	1

- In each of the load sets with exponentially distributed IAT's, the optimal solution was equal to the lower bound.
- In each of the load sets with uniformly distributed IAT's and high variability in the IAT's the lower bound on z^* was achieved.
- The multi-cycle lower bound was tight for all but 7 of the load sets: $U1$, $U2$, $U4$, $U5$, $LU2$, and $LU3$. For each of these 7 load sets, the multi-cycle lower bound differed from the optimal solution by 1 driver.
- The heuristics provided an optimal solution for all but 8 of the load sets: $U0$, $LU0$, $LU1$, $LU3$, $LU4$, $LU5$, $LU8$, $LU10$.
- The largest absolute difference between the heuristic upper bound and the optimal solution was 3 drivers.
- The largest relative difference between the heuristic upper bound and the optimal solution was 12 percent.
- The ratio for the cycle UB to the cycle LB ranged from 1.05 to 1.25.

2.9.3.4 18-Hour Lane, 5 Rest Facilities

Table 6 provides the results for the 18-hour lane with 5 rest facilities. The following are the key observations.

- All but 4 of the load sets were solved optimally in under 60 seconds. All were solved optimally in under 126 seconds.
- Among the load sets with uniformly distributed IAT's, more variability in the IAT's resulted in shorter run times but more required drivers.
- In each of the load sets with exponentially distributed IAT's, the optimal solution was equal to the lower bound.
- In each of the load sets with uniformly distributed IAT's and high variability in the IAT's the lower bound on z^* was achieved.

Table 6: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 18-Hour Lane with 5 Rest Facilities

Load Set	Λ_1	Λ_M	z^*	Ω_H	Ω_1	Run Time (sec)
$U0$	8	9	9	10	10	60
$U1$	9	9	10	10	11	13
$U2$	9	9	10	10	11	16
$U3$	10	10	10	10	11	14
$U4$	9	9	10	10	10	14
$U5$	9	9	10	10	11	23
$U6$	13	13	13	13	15	28
$U7$	14	14	14	14	15	16
$U8$	12	12	12	12	14	7
$U9$	13	13	13	13	15	4
$U10$	12	12	12	12	14	3
$LU0$	24	25	25	27	27	126
$LU1$	26	26	26	27	30	63
$LU2$	25	25	26	26	29	34
$LU3$	25	25	25	27	29	97
$LU4$	25	25	26	27	29	43
$LU5$	25	25	26	27	29	75
$LU6$	30	30	30	30	34	5
$LU7$	29	29	29	29	33	4
$LU8$	30	30	30	30	33	7
$LU9$	28	28	28	28	33	5
$LU10$	30	30	30	30	34	41
$E1$	20	20	20	20	21	13
$E2$	15	15	15	15	16	3
$E3$	19	19	19	19	21	38
$E4$	20	20	20	20	21	2
$E5$	18	18	18	18	19	16
$LE1$	40	40	40	40	44	4
$LE2$	35	35	35	35	39	4
$LE3$	39	39	39	39	42	3
$LE4$	37	37	37	37	39	2
$LE5$	38	38	38	38	41	14

- The multi-cycle lower bound was tight for all but 8 of the load sets: $U0$, $U1$, $U2$, $U4$, $U5$, $LU2$, $LU4$, and $LU5$. For each of these 8 load sets, the multi-cycle lower bound differed from the optimal solution by 1 driver.

- The heuristics provided an optimal solution for all but 6 of the load sets: $U0$, $LU0$, $LU1$, $LU3$, $LU4$, and $LU5$.

- The largest absolute difference between the heuristic upper bound and the optimal solution was 2 drivers.

- The largest relative difference between the heuristic upper bound and the optimal solution was 8 percent.

- The ratio for the cycle UB to the cycle LB ranged from 1.05 to 1.25.

2.9.3.5 18-Hour Lane, 11 Rest Facilities

Table 7 provides the results for the 18-hour lane with 11 rest facilities. The following are the key observations.

- All but 6 of the load sets were solved optimally in under 5 minutes. All were solved optimally in under 31 minutes.

- Among the load sets with uniformly distributed IAT's, more variability in the IAT's resulted in shorter run times but more required drivers.

- In each of the load sets with exponentially distributed IAT's, the optimal solution was equal to the lower bound.

- In each of the load sets with uniformly distributed IAT's and high variability in the IAT's the lower bound on z^* was achieved.

- For the load sets with uniformly distributed IAT's and low variability in the IAT's the lower bound on z^* was achieved for only 3 of the 10 load sets.

- For the load sets with uniformly distributed IAT's and no variability in the IAT's the lower bound on z^* was not achieved for either of the load sets.

Table 7: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 18-Hour Lane with 11 Rest Facilities

`Load Set`	Λ_1	Λ_M	z^*	Ω_H	Ω_1	`Run Time (sec)`
$U0$	8	9	9	10	10	495
$U1$	9	9	10	10	11	80
$U2$	9	9	10	10	11	118
$U3$	10	10	10	10	11	37
$U4$	9	9	10	10	10	90
$U5$	9	9	10	10	11	116
$U6$	13	13	13	13	15	207
$U7$	14	14	14	14	15	305
$U8$	12	12	12	12	14	41
$U9$	13	13	13	13	15	232
$U10$	12	12	12	12	14	21
$LU0$	24	25	25	27	27	1837
$LU1$	26	26	26	27	30	206
$LU2$	25	25	26	26	29	155
$LU3$	25	25	25	27	29	718
$LU4$	25	25	26	27	29	426
$LU5$	25	25	26	27	29	336
$LU6$	30	30	30	30	34	9
$LU7$	29	29	29	29	33	30
$LU8$	30	30	30	30	33	53
$LU9$	28	28	28	28	33	30
$LU10$	30	30	30	30	34	26
$E1$	20	20	20	20	21	165
$E2$	15	15	15	15	16	223
$E3$	19	19	19	19	21	152
$E4$	20	20	20	20	21	20
$E5$	18	18	18	18	19	139
$LE1$	40	40	40	40	44	148
$LE2$	35	35	35	35	39	26
$LE3$	39	39	39	39	42	164
$LE4$	37	37	37	37	39	9
$LE5$	38	38	38	38	41	112

- The multi-cycle lower bound was tight for all but 7 of the load sets: $U1$, $U2$, $U4$, $U5$, $LU2$, $LU4$, and $LU5$. For each of these 7 load sets, the multi-cycle lower bound differed from the optimal solution by 1 driver.

- The heuristics provided an optimal solution for all but 6 of the load sets: $U0$, $LU0$, $LU1$, $LU3$, $LU4$, and $LU5$.

- The largest absolute difference between the heuristic upper bound and the optimal solution was 2 drivers.

- The largest relative difference between the heuristic upper bound and the optimal solution was 8 percent.

- The ratio for the cycle UB to the cycle LB ranged from 1.05 to 1.25.

2.9.3.6 18-Hour Lane, 17 Rest Facilities (Unrestricted)

Table 8 provides the results for the 18-hour lane with 17 rest facilities. This number of facilities provides a rest facility at each location an unrestricted driver would stop to rest. Therefore, the results are the same as for an unrestricted driver. The following are the key observations.

- All but 9 of the load sets were solved optimally in under 5 minutes. Only 1 load set, $LU0$, required more than 30 minutes to solve optimally, requiring 1 hour and 44 minutes.

- Among the load sets with uniformly distributed IAT's, more variability in the IAT's resulted in shorter run times but more required drivers.

- In each of the load sets with exponentially distributed IAT's, the optimal solution was equal to the lower bound.

- In each of the load sets with uniformly distributed IAT's and high variability in the IAT's the lower bound on z^* was achieved.

Table 8: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 18-Hour Lane with 17 Rest Facilities (Unrestricted)

Load Set	Λ_1	Λ_M	z^*	Ω_H	Ω_1	Run Time (sec)
$U0$	8	9	9	10	10	1600
$U1$	9	9	10	10	11	703
$U2$	9	9	10	10	11	468
$U3$	10	10	10	10	11	107
$U4$	9	9	10	10	10	818
$U5$	9	9	10	10	11	756
$U6$	13	13	13	13	15	213
$U7$	14	14	14	14	15	253
$U8$	12	12	12	12	14	244
$U9$	13	13	13	13	15	230
$U10$	12	12	12	12	14	174
$LU0$	24	25	25	27	27	6280
$LU1$	26	26	26	27	30	332
$LU2$	25	25	26	26	29	790
$LU3$	25	25	25	27	29	1450
$LU4$	25	25	26	27	29	600
$LU5$	25	25	26	27	29	448
$LU6$	30	30	30	30	34	108
$LU7$	29	29	29	29	33	295
$LU8$	30	30	30	30	33	132
$LU9$	28	28	28	28	33	110
$LU10$	30	30	30	30	34	92
$E1$	20	20	20	20	21	137
$E2$	15	15	15	15	16	224
$E3$	19	19	19	19	21	96
$E4$	20	20	20	20	21	291
$E5$	18	18	18	18	19	62
$LE1$	40	40	40	40	44	173
$LE2$	35	35	35	35	39	68
$LE3$	39	39	39	39	42	195
$LE4$	37	37	37	37	39	86
$LE5$	38	38	38	38	41	152

- For the load sets with uniformly distributed IAT's and low variability in the IAT's the lower bound on z^* was achieved for only 3 of the 10 load sets.

- For the load sets with uniformly distributed IAT's and no variability in the IAT's the lower bound on z^* was not achieved for either of the load sets.

- The multi-cycle lower bound was tight for all but 7 of the load sets: $U1$, $U2$, $U4$, $U5$, $LU2$, $LU4$, and $LU5$. For each of these 7 load sets, the multi-cycle lower bound differed from the optimal solution by 1 driver.

- The heuristics provided an optimal solution for all but 6 of the load sets: $U0$, $LU0$, $LU1$, $LU3$, $LU4$, and $LU5$.

- The largest absolute difference between the heuristic upper bound and the optimal solution was 2 drivers.

- The largest relative difference between the heuristic upper bound and the optimal solution was 8 percent.

- The ratio for the cycle UB to the cycle LB ranged from 1.05 to 1.25.

2.9.3.7 9-Hour Lane, 1 Rest Facility

Table 9 provides the results for the 9-hour lane with 1 rest facility. The following are the key observations.

- Each of the load sets with the exception of LU0 through LU5 were solved optimally in under 20 seconds. The longest run time was 22 minutes.

- Among the load sets with uniformly distributed IAT's, more variability in the IAT's resulted in shorter run times but more required drivers.

- In each of the load sets with exponentially distributed IAT's, optimal solution was equal to the lower bound.

- For the load sets with uniformly distributed IAT's, higher mean IAT's and more variability in the IAT's made achieving the lower bound on z* more likely.

Table 9: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 9-Hour Lane with 1 Rest Facility

Load Set	Λ_1	Λ_M	z^*	Ω_H	Ω_1	Run Time (sec)
$U0$	4	5	5	5	5	8
$U1$	4	5	6	6	6	8
$U2$	5	5	6	6	6	6
$U3$	5	5	6	6	6	5
$U4$	4	5	6	6	6	7
$U5$	4	5	6	6	6	8
$U6$	8	8	8	8	10	1
$U7$	8	8	8	8	9	1
$U8$	7	7	7	7	8	2
$U9$	8	8	8	8	10	1
$U10$	7	7	7	7	8	1
$LU0$	10	14	14	14	14	116
$LU1$	12	14	15	15	15	571
$LU2$	11	14	14	15	15	1309
$LU3$	11	14	14	14	15	45
$LU4$	11	14	14	15	15	90
$LU5$	11	14	14	15	15	301
$LU6$	14	15	16	16	19	4
$LU7$	14	15	15	15	18	4
$LU8$	14	15	15	16	19	17
$LU9$	14	15	15	15	18	4
$LU10$	14	16	16	17	19	2
$E1$	12	12	12	12	15	1
$E2$	10	10	10	10	11	1
$E3$	12	12	12	12	15	1
$E4$	12	12	12	12	14	1
$E5$	10	10	10	10	12	1
$LE1$	21	21	21	21	28	1
$LE2$	20	20	20	20	24	1
$LE3$	20	20	20	20	27	1
$LE4$	20	20	20	20	25	1
$LE5$	21	21	21	22	26	1

- The multi-cycle lower bound was tight for all but 7 of the load sets: $U1$, $U2$, $U3$, $U4$, $U5$, $LU1$, and $LU6$. For each of these 7 load sets, the multi-cycle lower bound differed from the optimal solution by 1 driver.
- The heuristics provided an optimal solution for all but 5 of the load sets: $LU2$, $LU4$, $LU5$, $LU8$, and $LU10$.
- The largest absolute difference between the heuristic upper bound and the optimal solution was 1 driver.
- The largest relative difference between the heuristic upper bound and the optimal solution was 7 percent.
- The ratio for the cycle UB to the cycle LB ranged from 1.1 to 1.5.

2.9.3.8 9-Hour Lane, 2 Rest Facilities

Table 10 provides the results for the 9-hour lane with 2 rest facilities. The following are the key observations.

- Each of the load sets with the exception of $LU0$ and $LU5$ were solved optimally in under 30 seconds. The longest run time was 68 seconds.
- Among the load sets with uniformly distributed IAT's and the lower mean IAT, more variability in the IAT's resulted in shorter run times but more required drivers. When the mean IAT was 500, this difference in run times between low and high variability was not observed.
- In each of the load sets with exponentially distributed IAT's, the optimal solution was equal to the lower bound.
- For the load sets with uniformly distributed IAT's, higher mean IAT's and more variability in the IAT's made achieving the lower bound on z* more likely.
- The multi-cycle lower bound was tight for all but 5 of the load sets: $LU2$, $LU3$, $LU4$, $LU5$, and $LU6$. For each of these 5 load sets, the multi-cycle lower bound differed from the optimal solution by 1 driver.

Table 10: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 9-Hour Lane with 2 Rest Facilities

Load Set	Λ_1	Λ_M	z^*	Ω_H	Ω_1	Run Time (sec)
$U0$	4	5	5	5	5	11
$U1$	4	5	5	5	6	9
$U2$	5	5	5	6	6	4
$U3$	5	5	5	5	6	4
$U4$	4	5	5	5	6	7
$U5$	4	5	5	6	6	10
$U6$	8	8	8	8	10	7
$U7$	8	8	8	8	9	14
$U8$	7	7	7	7	8	6
$U9$	8	8	8	8	10	16
$U10$	7	7	7	7	8	3
$LU0$	10	13	13	14	14	68
$LU1$	12	14	14	15	15	17
$LU2$	11	13	14	14	15	22
$LU3$	11	13	14	14	15	22
$LU4$	11	13	14	15	15	26
$LU5$	11	13	14	14	15	42
$LU6$	14	15	16	16	19	5
$LU7$	14	15	15	16	18	5
$LU8$	14	15	15	16	19	23
$LU9$	14	15	15	15	18	5
$LU10$	14	16	16	16	19	3
$E1$	12	12	12	12	15	5
$E2$	10	10	10	10	11	4
$E3$	12	12	12	12	15	8
$E4$	12	12	12	12	14	2
$E5$	10	10	10	10	12	2
$LE1$	21	21	21	21	28	2
$LE2$	20	20	20	20	24	2
$LE3$	20	20	20	21	27	2
$LE4$	20	20	20	20	25	2
$LE5$	21	21	21	21	26	2

- The heuristics provided an optimal solution for all but 8 of the load sets: $U2$, $U5$, $LU0$, $LU1$, $LU4$, $LU7$, $LU8$ and $LE3$.
- The largest absolute difference between the heuristic upper bound and the optimal solution was 1 driver.
- The largest relative difference between the heuristic upper bound and the optimal solution was 20 percent.
- The ratio for the cycle UB to the cycle LB ranged from 1.1 to 1.5.

2.9.3.9 9-Hour Lane, 3 Rest Facilities

Table 11 provides the results for the 9-hour lane with 3 rest facilities. The following are the key observations.

- One load set, $LU4$, could not be solved optimally in the 2 hour run time limit.
- Each of the load sets with the exception of $LU4$ were solved optimally in under 16 minutes.
- Among the load sets with uniformly distributed IAT's and the lower mean IAT, more variability in the IAT's resulted in shorter run times but more required drivers. When the mean IAT was 500, the difference in run times between low and high variability was not as clear.
- Among the load sets with exponentially distributed IAT's, load sets with lower mean IAT's took longer to solve on average.
- In each of the load sets with exponentially distributed IAT's, the optimal solution was equal to the lower bound.
- For the load sets with uniformly distributed IAT's, higher mean IAT's and more variability in the IAT's made achieving the lower bound on z^* more likely.
- The multi-cycle lower bound was tight for all but 2 of the load sets: $LU9$ and $LU10$. Tightness of the bound for $LU4$ could not be determined because the optimal solution

Table 11: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 9-Hour Lane with 3 Rest Facilities

Load Set	Λ_1	Λ_M	z^*	Ω_H	Ω_1	Run Time (sec)
$U0$	4	5	5	5	5	61
$U1$	4	5	5	5	6	45
$U2$	5	5	5	5	6	21
$U3$	5	5	5	5	6	37
$U4$	4	5	5	5	6	25
$U5$	4	5	5	6	6	81
$U6$	8	8	8	8	10	110
$U7$	8	8	8	8	9	57
$U8$	7	7	7	7	8	11
$U9$	8	8	8	8	10	10
$U10$	7	7	7	7	8	90
$LU0$	10	13	13	14	14	946
$LU1$	12	13	14	14	15	358
$LU2$	11	13	14	14	15	286
$LU3$	11	13	13	14	15	465
$LU4$	11	13	**	14	15	7200
$LU5$	11	13	13	14	15	568
$LU6$	14	15	15	16	19	73
$LU7$	14	15	15	16	18	38
$LU8$	14	15	15	15	19	68
$LU9$	14	14	15	15	18	69
$LU10$	14	15	16	16	19	58
$E1$	12	12	12	12	15	52
$E2$	10	10	10	10	11	51
$E3$	12	12	12	12	15	35
$E4$	12	12	12	12	14	58
$E5$	10	10	10	10	12	65
$LE1$	21	21	21	21	28	70
$LE2$	20	20	20	20	24	10
$LE3$	20	20	20	21	27	100
$LE4$	20	20	20	20	25	150
$LE5$	21	21	21	21	26	53

** Optimal solution not found within computational time limit of 2 hours.
Best integer solution at termination was 14.

was not found. However, for each of these 3 load sets, the multi-cycle lower bound differed from the optimal solution by no more than 1 driver.

- The heuristics provided an optimal solution for all but 8 of the load sets: $U5$, $LU0$, $LU3$, $LU4$, $LU5$, $LU6$, $LU7$, and $LE3$.
- The largest absolute difference between the heuristic upper bound and the optimal solution was 1 driver.
- The largest relative difference between the heuristic upper bound and the optimal solution was 20 percent.
- The ratio for the cycle UB to the cycle LB ranged from 1.1 to 1.5.

2.9.3.10 9-Hour Lane, 8 Rest Facilities (Unrestricted)

Table 12 provides the results for the 9-hour lane with 8 rest facilities. This number of facilities provides a rest facility at each location an unrestricted driver would stop to rest. Therefore, the results are the same as for an unrestricted driver. The following are the key observations.

- One load set, $LU4$, could not be solved optimally in the 2 hour run time limit.
- All but 4 of the load sets could be solved optimally in under 20 minutes.
- Among the load sets with uniformly distributed IAT's, more variability in the IAT's resulted in shorter run times but more required drivers. The difference in run times was more pronounced for shorter mean IAT's.
- In each of the load sets with exponentially distributed IAT's, the optimal solution was equal to the lower bound.
- For the load sets with uniformly distributed IAT's, higher mean IAT's and more variability in the IAT's made achieving the lower bound on z^* more likely.
- The multi-cycle lower bound was tight for all but 4 of the load sets: $LU1$, $LU2$, $LU9$, and $LU10$. Tightness of the bound for $LU4$ could not be determined because the

Table 12: Bounds, Run Times and Optimal Number of Drivers by Load Set for the 9-Hour Lane with 8 Rest Facilities (Unrestricted)

Load Set	Λ_1	Λ_M	z^*	Ω_H	Ω_1	Run Time (sec)
$U0$	4	5	5	5	5	707
$U1$	4	5	5	5	6	680
$U2$	5	5	5	5	6	460
$U3$	5	5	5	5	6	201
$U4$	4	5	5	5	6	300
$U5$	4	5	5	6	6	706
$U6$	8	8	8	8	10	264
$U7$	8	8	8	8	9	270
$U8$	7	7	7	7	8	225
$U9$	8	8	8	8	10	210
$U10$	7	7	7	7	8	282
$LU0$	10	13	13	14	14	3588
$LU1$	12	13	14	14	15	1100
$LU2$	11	13	14	14	15	1076
$LU3$	11	13	13	14	15	4620
$LU4$	11	13	$**$	14	15	7200
$LU5$	11	13	13	14	15	3540
$LU6$	14	15	15	16	19	431
$LU7$	14	15	15	16	18	291
$LU8$	14	15	15	15	19	802
$LU9$	14	14	15	15	18	425
$LU10$	14	15	16	16	19	701
$E1$	12	12	12	12	15	200
$E2$	10	10	10	10	11	172
$E3$	12	12	12	12	15	188
$E4$	12	12	12	12	14	133
$E5$	10	10	10	10	12	152
$LE1$	21	21	21	21	28	244
$LE2$	20	20	20	20	24	164
$LE3$	20	20	20	21	27	56
$LE4$	20	20	20	20	25	261
$LE5$	21	2121	21	26	152	

`** Optimal solution not found within computational time limit of 2 hours.`
`Best integer solution at termination was 14.`

Table 13: 18-Hour Lane Average Run Times by Number of Rest Facilities and Load Set Type (Seconds)

IAT Distr	Mean IAT	IAT Variability	2RF	3RF	4RF	5RF	11RF	Unrestricted
Uniform	170	None	18	4	33	126	1837	6280
Uniform	170	Low	30.4	1.4	12.6	62.4	368.2	724
Uniform	170	High	2	1	1.6	12.4	29.6	147.4
Uniform	500	None	104	2	33	72	495	1600
Uniform	500	Low	4.2	1.8	4.4	16	88.2	570.4
Uniform	500	High	1	1	3	11.6	173.2	222.8
Exponential	170	NA	1	1	2	14.4	139.8	162
Exponential	500	NA	1	1	1.4	5.4	91.8	134.8

optimal solution was not found. However, for each of these 5 load sets, the multi-cycle lower bound differed from the optimal solution by no more than 1 driver.

- The heuristics provided an optimal solution for all but 8 of the load sets: $U5$, $LU0$, $LU3$, $LU4$, $LU5$, $LU6$, $LU7$, and $LE3$.

- The largest absolute difference between the heuristic upper bound and the optimal solution was 1 driver.

- The largest relative difference between the heuristic upper bound and the optimal solution was 20 percent.

- The ratio for the cycle UB to the cycle LB ranged from 1.1 to 1.5.

2.9.3.11 Results Summary and Discussion

A summary of average run times for the different load set characteristics on the 18-hour lane is presented in Table 13 and illustrated in Figures 7 and 8.

As Figures 7 and 8 clearly show, with the exception of the 2 rest facility case, run times increase exponentially with each increase in rest facilities. This is expected since the driver state space and the resulting number of nodes and arcs in the network formulation will increase as rest facilities are added.

Table 14 shows a summary of the average number of drivers needed for the different load set characteristics on the 18-hour lane. As this table shows, using more that 2 rest facilities provides no reduction in the number of drivers for the load sets with exponentially

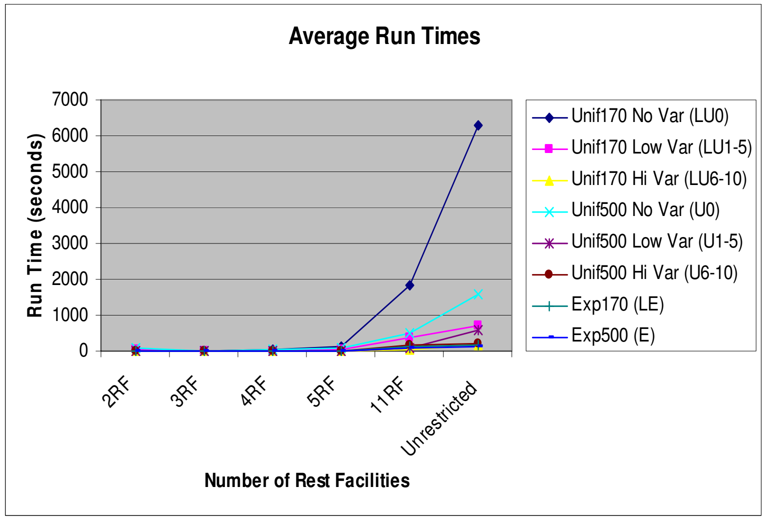

Figure 7: Graph of 18-Hour Lane Average Run Times by Number of Rest Facilities (All) and Load Set Type

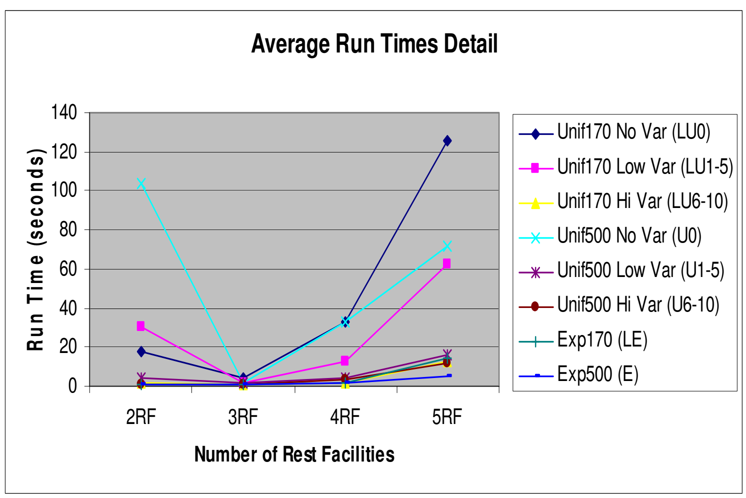

Figure 8: Graph of 18-Hour Lane Average Run Times by Number of Rest Facilities (2-5) and Load Set Type

Table 14: 18-Hour Lane Average Number of Drivers by Number of Rest Facilities and Load Set Type

IAT Distr	Mean IAT	IAT Variability	1RF	2RF	3RF	4RF	5RF	11RF	17RF
Uniform	170	None	27	27	26	26	25	25	25
Uniform	170	Low	29.2	27.8	27	26.2	25.8	25.8	25.8
Uniform	170	High	33.4	29.8	29.6	29.4	29.4	29.4	29.4
Uniform	500	None	10	10	10	9	9	9	9
Uniform	500	Low	10.8	10	10	10	10	10	10
Uniform	500	High	14.6	12.8	12.8	12.8	12.8	12.8	12.8
Exponential	170	NA	41	37.8	37.8	37.8	37.8	37.8	37.8
Exponential	500	NA	19.6	18.4	18.4	18.4	18.4	18.4	18.4

Table 15: 9-Hour Lane Average Run Times by Number of Rest Facilities and Load Set Type (Seconds)

IAT Distr	Mean IAT	IAT Variability	1RF	2RF	3RF	Unrestricted
Uniform	170	None	116	68	946	3588
Uniform	170	Low	463.2	25.8	1775.4	3507.2
Uniform	170	High	6.2	8.2	61.2	446.6
Uniform	500	None	8	11	61	707
Uniform	500	Low	6.8	6.8	41.8	469.4
Uniform	500	High	1.2	9.2	55.6	250.2
Exponential	170	NA	1	2	76.6	175.4
Exponential	500	NA	1	4.2	52.2	171

distributed IAT's. The load sets with uniformly distributed IAT's generally require more rest facilities before no improvement is observed. Also note that when reducing the mean IAT by a third, load sets with uniformly distributed IAT's required 2.5 to 3 times the number of drivers, whereas the load sets with exponentially distributed IAT's required only twice the number of drivers.

A summary of average run times for the different load set characteristics on the 9-hour lane is presented in Table 15 and illustrated in Figures 9 and 10. Figure 10 eliminates the Uniform 170 no variation and low variation parameter sets to allow a clearer representation of the other load set parameters. As these graphs clearly illustrate, run times increase exponentially with increases in rest facilities due to the growth of the network from the larger number of driver states.

Table 16 shows a summary of the average number of drivers needed for the different load set characteristics on the 9-hour lane. As this table shows, using more that 1 rest

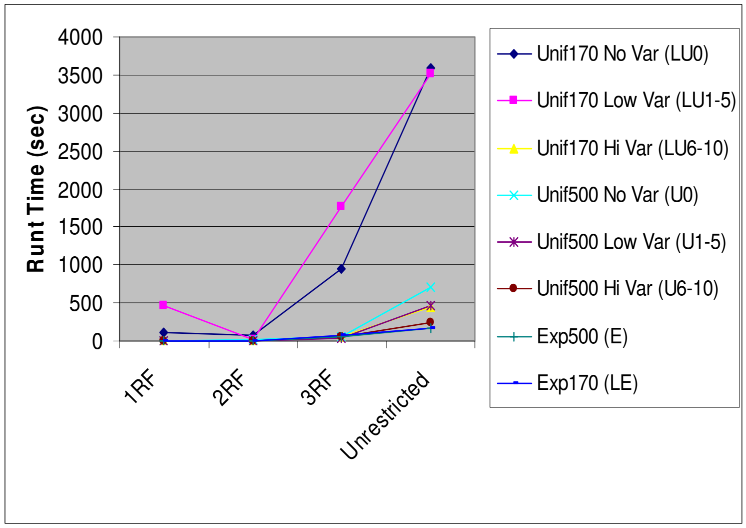

Figure 9: Graph of 9-Hour Lane Average Run Times by Number of Rest Facilities and Load Set Type (All)

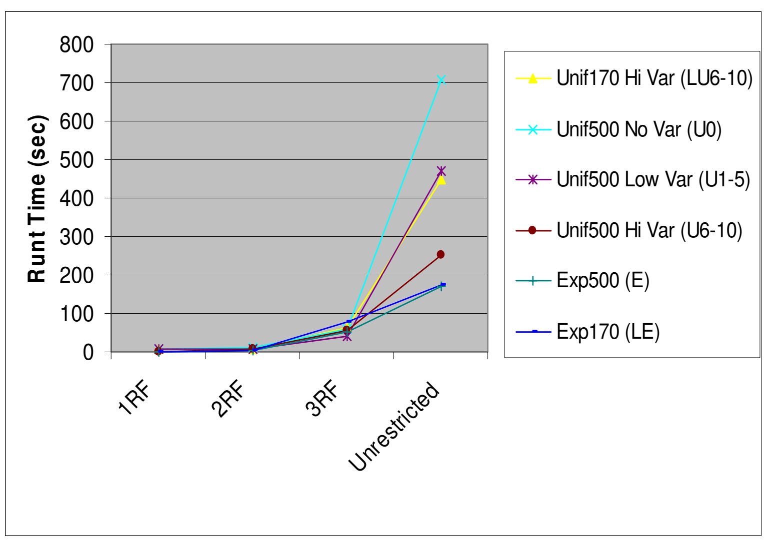

Figure 10: 9-Hour Lane Average Run Times by Number of Rest Facilities and a Subset of the Load Set Types

Table 16: 9-Hour Lane Average Number of Drivers by Number of Rest Facilities and Load Set Type

IAT Distr	Mean IAT	IAT Variability	0RF	1RF	2RF	3RF	Unrestricted
Uniform	170	None	14	14	13	13	13
Uniform	170	Low	15	14.2	14	13.6	13.6
Uniform	170	High	18.6	15.4	15.4	15.2	15.2
Uniform	500	None	5	5	5	5	5
Uniform	500	Low	6	6	5	5	5
Uniform	500	High	9	7.6	7.6	7.6	7.6
Exponential	170	NA	26	20.4	20.4	20.4	20.4
Exponential	500	NA	13.4	11.2	11.2	11.2	11.2

facility provides no reduction in the number of drivers for the load sets with exponentially distributed IAT's. The load sets with uniformly distributed IAT's generally require more rest facilities before no improvement is observed. Also note that when reducing the mean IAT by a third, load sets with uniformly distributed IAT's required 2.5 to 3 times the number of drivers, whereas the load sets with exponentially distributed IAT's required only twice the number of drivers.

Some general observations from the results:

1. Of the load sets analyzed, load sets with exponentially distributed IAT's were the easiest to solve to optimality and also achieved the lower bound on the number of drivers using fewer rest facilities than the load sets with the uniformly distributed IAT's.

2. The heuristics provided optimal solutions more frequently for the load sets with exponentially distributed IAT's than the load sets with the uniformly distributed IAT's.

3. Increasing variability in the IAT's tends to reduce the required run time to find the optimal solution.

4. Increasing variability in the IAT's tends to require a higher number of drivers.

5. Increasing the number of driver states (by increasing rest facilities) tends to increase the required run time to find the optimal solution.

6. The multi-cycle lower bound is tight in more than 83% of the instances analyzed, and within one driver of the optimal in the remaining instances.

The relative difference between upper and lower bounds on the number of drivers will depend upon the lane length and the resulting cycle times, CT_{min} and CT_{max}. In this experiment, $\frac{CT_{max}}{CT_{min}} = \frac{76}{66} = 1.15$ for the 18-hour lane and $\frac{CT_{max}}{CT_{min}} = \frac{38}{28} = 1.36$ for the 9-hour lane. Since the bounds on the number of drivers are determined by determining the maximum number of loads within windows of CT_{min} and CT_{max} lengths, we would expect a strong correlation between $\frac{CT_{max}}{CT_{min}}$ and $\frac{\Omega_1}{\Lambda_1}$. This correlation was confirmed by this experiment. The significance of this result is that for longer lanes, the differences between the minimum and maximum number of drivers will be relatively small and the effect of restricting driver rest to only designated facilities will be small as well.

The load dispatch time characteristics have perhaps the most influence on this problem. Note that the load sets with uniformly distributed IAT's required additional rest facilities to achieve the lower bound number of drivers and in some cases, the lower bound could not be achieved. The load sets with exponentially distributed IAT's, on the other hand, could achieve the lower bound with very few rest facilities.

For the 9-hour lane, determining z^* could not be done within the 2-hour run-time limit for 1 load set instance for 2 of the rest facility configurations. In both of these cases, however, the difference between the multi-cycle lower bound and the best heuristic solution was a single driver. Using the heuristic solution in this case would have little if any detrimental effect on dispatching efficiency.

Using a single rest facility on the 18-hour lane required up to 5 additional drivers or at most an 18% increase over the unrestricted rest case. Only 1 of the load sets required no additional drivers over the unrestricted rest case when using a single rest facility. When using 2 rest facilities, no additional drivers were required for 23 of the 42 load sets and no more than 2 additional drivers or no more than 11% additional drivers were required on any of the other load sets. Using 5 rest facilities required no additional drivers over the unrestricted rest case for all load sets analyzed.

Using no facilities on the 9-hour lane required up to 7 additional drivers, or at most a 35%

Table 17: Heuristic Performance by Lane Length and IAT Distribution

Lane Length	IAT Distr	Instances With Heuristic Optimal	Number of Instances	Percent Optimal
18	Uniform	93	132	70.5
	Exponential	59	60	98.3
	Total	152	192	79.2
9	Uniform	62	88	70.5
	Exponential	36	40	90.0
	Total	98	128	76.6
Overall	Uniform	155	220	70.5
	Exponential	95	100	95.0
	Total	250	320	78.1

increase over the unrestricted rest case, with only 6 of the 42 load sets requiring no additional drivers. Adding only 1 rest facility to the 9-hour lane provided significant improvement, requiring at most 1 additional driver, or up to a 20% increase over the unrestricted rest case, with 23 of the 42 load sets requiring no additional drivers. Using 2 rest facilities resulted in 39 of the 42 load sets requiring no additional drivers, with at most 1 additional driver or up to an 8% increase in the number of drivers needed compared to the unrestricted rest case. Using 3 rest facilities required no additional drivers over the unrestricted rest case for all load sets analyzed.

2.9.3.12 Heuristic Performance

The overall performance of the heuristics on all instances is summarized in Table 17. The heuristics performed better on load sets with exponentially distributed IAT's, producing an optimal solution for 95% of the instances. Among these instances, the heuristics performed slightly better on the 18-hour lane than on the 9-hour lane. For the load sets with uniformly distributed IAT's, the heuristics produced optimal solutions in 70.5% of the instances for both the 9-hour lane and the 18-hour lane. What is not indicated by the table is that in the cases where the heuristics were not optimal, the heuristic solution differed from the optimal solution by only a single driver in 58 instances, by 2 drivers in 9 instances and by 3 drivers in 3 instances. For the 9-hour lane, the heuristic solution was within 1 driver of the optimal solution for every load set instance. In summary, the heuristics were very effective at generating good solutions in almost all cases.

Looking at the heuristics a little closer, some of the heuristics were dominated by others. For the 18-hour lane:

- Highest State LIFO was dominated by Load Density Cycle Time.
- Lowest Unrested State was dominated by Lowest Cycle Time with LIFO Tiebreaker.
- Highest State was dominated by Lowest Cycle Time with LIFO Tiebreaker.
- LIFO was dominated by Lowest Cycle Time with LIFO Tiebreaker.
- Lowest State was dominated by Lowest State LIFO.
- Lowest Index was dominated by Lowest State LIFO.
- FIFO was dominated by Lowest State LIFO.
- Lowest Cycle Time was dominated by Lowest State LIFO.

The following 3 heuristics were not dominated by any others for the 18-hour lane instances:

- Lowest State with LIFO Tiebreaker.
- Lowest Cycle Time with LIFO Tiebreaker.
- Load Density Cycle Time.

For the 9-hour lane:

- Highest State was dominated by Lowest Cycle Time LIFO.
- LIFO was dominated by Lowest Cycle Time LIFO.
- Lowest State LIFO was dominated by Lowest Cycle Time LIFO.
- Lowest State was dominated by Lowest Cycle Time LIFO.
- Lowest Index was dominated by Lowest Cycle Time LIFO.
- Lowest Cycle Time was dominated by Lowest Cycle Time LIFO.

The following 5 heuristics were not dominated by any others for the 9-hour lane instances:

- FIFO.
- Highest State with LIFO Tiebreaker.
- Lowest Unrested State.
- Lowest Cycle Time with LIFO Tiebreaker.
- Load Density Cycle Time.

The only two heuristics that were not dominated in either lane length were the *Lowest Cycle Time with LIFO Tiebreaker* and the *Load Density Cycle Time* heuristics. The 5 heuristics that were dominated for all instances in both lane lengths (*Highest State, LIFO, Lowest State, Lowest Index*, and *Lowest Cycle Time*) could probably be eliminated from consideration without any detrimental effects, but while certain heuristics performed better empirically, there is no guarantee that they will always perform better. As an illustration, consider the Lowest state with LIFO tiebreaker heuristic which was non-dominated in the 18-hour lane instances.

Consider an 18-hour lane with 2 rest facilities located such that the driver cycle times are 66 hours on the first load and 76 hours on every subsequent load. Consider a load set with load pickup times of 0, 0, 0, 100, 101, 167, 170, 173, 244, 247, and 248 hours. The Lowest State LIFO heuristic would assign these loads to the drivers as follows:

- Time 0 - driver 1 - CT 66
- Time 0 - driver 2 - CT 66
- Time 0 - driver 3 - CT 66
- Time 100 - driver 1 - CT 66
- Time 101 - driver 2 - CT 66
- Time 167 - driver 3 (Lowest State of the 3 available drivers) - CT 66

- Time 170 - driver 2 (LIFO of the 2 available drivers in the second state) - CT 76
- Time 173 - driver 1 (Only available driver)- CT 76
- Time 244 - driver 3 (Only available driver)- CT 76
- Time 247 - driver 2 (Only available driver)- CT 76
- Time 248 - driver 4 (1,2,3 are unavailable)

This assignment requires 4 drivers. Now consider the Lowest Index heuristic:

- Time 0 - driver 1 - CT 66
- Time 0 - driver 2 - CT 66
- Time 0 - driver 3 - CT 66
- Time 100 - driver 1 - CT 66
- Time 101 - driver 2 - CT 66
- Time 167 - driver 1 - CT 76
- Time 170 - driver 2 - CT 76
- Time 173 - driver 3 - CT 66
- Time 244 - driver 1 - CT 76
- Time 247 - driver 2 - CT 76
- Time 248 - driver 3 - CT 76

This assignment only requires 3 drivers. In this case, reusing driver 1 at time 167 allowed driver 3 to remain fully rested when assigned the time 173 load, making him available for the time 248 load.

The lowest cycle time with LIFO tiebreaker also dominated the lowest cycle time heuristic but this is not guaranteed to be the case since drivers with identical cycle times may be in different states. While assigning the most recently returned driver still allows the other drivers a higher probability of reaching a fully rested state before their next assignment, the subsequent state of the driver assigned is state dependent. For example, for a cycle time sequence of 66,66, then 76 which repeats, drivers in both of the first two states have the same cycle time, but assigning the driver in the second state will mean his subsequent cycle time will be 76 hours, whereas assigning the driver in the first state will will mean his subsequent cycle time will be 66 hours. Therefore, which driver to assign this load in order to use the fewest drivers could depend on the timing of subsequent loads. One set of loads which illustrates this are pick up times of 200, 202, 266, 268, 332, 344 and two drivers who returned at 198 and 199, one in the first state, the other in the second state. In order for these loads to be feasibly delivered by these two drivers, the driver in the first states MUST be assigned the first load. If this is the lowest index driver, and the higher index driver is the most recently returned in the second state, then the LIFO tiebreaker performs worse than the lowest index tiebreaker, requiring 3 drivers instead of 2.

2.10 Contributions

We now summarize the primary contributions of the research reported in this chapter.

- Developed an approach for estimating the impact of restricting driver rest to designated locations on driver productivity.
- Contributions relating to bounds on the number of drivers:
  - Developed a new multi-cycle lower bound on the minimum number of drivers required for a given dispatching problem, where the previously established cycle bound is a special case. Preliminary computational evidence indicates that this multi-cycle bound is tight in more than 83% of the instances analyzed, and within one driver of the optimal in the remaining instances.
  - Proved that z^* can never be more than 2 times the multi-cycle lower bound for lane lengths longer than $\tau_R/2$. Demonstrated an instance in which $z^* = 2\Lambda_m$.
  - Proved an upper bound on the ratio of the single cycle upper bound to the single cycle lower bound for any lane length.

- Contributions relating to solution methods:
  - Developed a network flow formulation with bundling constraints for the minimum driver to load assignment problem which produces solutions in a reasonable amount of time for most scenarios.
  - Showed that a small set of dispatching heuristics perform very well empirically for matching drivers to loads, yielding solutions with numbers of drivers minimal or nearly minimal in all instances.
  - Proved that these heuristics will provide a solution using no more than $2z^*$ for any load set instance on lane lengths longer than $\tau_R/2$.
- Showed that in the single 18-hour lane case, restricting rest to 2 or 3 designated facilities is likely to have minimal impact on dispatching efficiency.
  - Using only 2 rest facilities required no additional drivers over the unrestricted rest case for all load set instances with exponentially distributed IAT's and some instances with uniformly distributed IAT's.
  - Using only 2 rest facilities resulted in less than an 11% increase for the remaining load sets with uniformly distributed IAT's.
- Showed that in the single 9-hour lane case, restricting rest to 1 or 2 designated facilities is likely to have minimal impact on dispatching efficiency.
  - Using only 1 rest facility required no additional drivers over the unrestricted rest case for all load set instances with exponentially distributed IAT's and most of the instances with uniformly distributed IAT's.
  - Using only 1 rest facility resulted in at most a 1 driver increase which is a relative increase of 6% to 20%.

CHAPTER III

THE REST FACILITY LOCATION PROBLEM

3.1 Introduction

In addition to determining the minimum number of drivers required for a given configuration of rest facilities, it is also natural to examine how to locate a fixed number of rest locations in order to minimize required driver fleet size. This chapter will begin with the simplest case, a single lane with no backhauls, develop insights into the nature of the rest facility location problem and present a mixed integer program that optimizes the rest facility locations.

We will then extend the insights and methods to various networks with multiple lanes.

3.2 The Single Lane No Backhaul Case

3.2.1 Possible Solution Methods

Several potential solution approaches for the optimal rest facility location problem will be explored briefly in this section. In this section, we assume that $\tau_D = 11$ and $\tau_R = 10$.

One approach is to place facilities so that driving time is maximized in each driving day. This method presents several difficulties. First, how many driving days should be analyzed? A possibility would be to maximize total travel time after any possible number of driving days. However, this would result in infeasible problems in many cases, and suboptimal results in other cases. Consider a 9-hour lane with 1 rest facility. Maximizing travel distance after 1 driving day would require the rest facility to be 7 hours from A so that the driver could drive 11 hours on the first day. However, this would result in only 18 total travel hours after 2 days, since the driver would be required to rest at A. Placing the facility at 2 hours from A instead results in only 9 hours driving in the first day, but allows for 11 in the second day, so that total travel time after 2 days is 20 hours, the maximum possible. This is in fact the optimal solution. The proposed method, however, would consider the problem to be infeasible since a different configuration is required for the total travel time after the first and second days.

This may indicate that maximizing total travel distance for some large number of travel days may lead to rest facility locations that are generally better, but the same 9-hour lane example shows that this approach may produce suboptimal results. If an odd number of driving days is used, this method would choose the 7-hour location, but if an even number of driving days is chosen, the 2-hour location would be chosen. Such inconsistency makes this an unsuitable solution method.

Another possible approach would be to place the rest facilities to maximize the number of fully utilized 11-hour driving periods for some given number of driving days. Like the previous method, however, the question is: how many driving days should be used in the analysis? Using the same 9-hour lane discussed above, if an even number of days were used, both the 2-hour location and the 7-hour location would result in one 11-hour driving period during every 2 driving days and this method would be indifferent between these two configurations. Alternatively, if an odd number of driving days were used, the 7-hour lane would be preferred, which is in fact a suboptimal solution.

In an optimal solution, the facilities must be located to minimize the time until a driver will be available after driving any number of cycles; thus, the driver returns to A more frequently, and is available for dispatch on the largest number of possible loads. A proper formulation must therefore minimize the total number of rests needed in total to complete any number of cycles, giving clear preference to reducing the number of rests in an earlier cycle over reducing the number of rests in later cycles. The mixed integer programming approach presented later in this chapter is designed to identify such solutions.

3.2.2 Key Concepts and Definitions

Definition 3.1. *An* ***optimal configuration*** *of k rest facilities on a given lane is a set of locations for those k rest facilities which results in the fewest required number of drivers to deliver any load set.*

A given rest area configuration will result in a specific driver cycle time sequence. In order to compare rest area configurations, a definition is needed that allows the determination of a preference order on cycle time sequences.

Definition 3.2. *Let $CT_{i\alpha}$ be the cycle time of the i^{th} cycle in the cycle time sequence α. Then a cycle time sequence α is* ***preferred*** *to cycle time sequence β if and only if:*

$$\sum_{i=1}^{n} CT_{i\alpha} \leq \sum_{i=1}^{n} CT_{i\beta} \tag{21}$$

for all n, and strict preference requires that the expression holds with strict inequality for at least one value of n.

This definition implies that any load set which can be feasibly delivered by a single driver under any other cycle time sequence can also be delivered by a single driver under the preferred cycle time sequence. This will allow any driver using a preferred cycle time sequence to deliver more loads in any given time frame. Note that while Definition 3.2 appears to imply the necessity of checking conditions (21) for an infinite number of cycles, this will not be the case in practice. More discussion of this point will come later in the chapter.

Since rest facility locations only affect the sequence of cycle times that each driver might experience between consecutive load dispatches at A, it should be possible to determine optimal rest area locations independent of the set of loads to be delivered. Thus, when comparing two sets of k rest facility locations, the set of locations that results in the preferred cycle time sequence is preferred.

As a simple illustration, consider a lane where $\delta_{AB} = 14$ with a single rest facility located at the halfway point ($d_1 = 7$). This location results in a cycle time sequence with a single repeating cycle time containing 4 rests. One way to reduce the number of rests in the cycle would be to eliminate the rest at B. Feasibility thus requires the facility to be within $\tau_D/2$ of B ($d_1 \geq 9.5$) and within τ_D of A ($d_1 \leq 11$). Choosing a location in the range 9.5 to 11 will reduce the cycle time to include only 3 rests on every cycle. To allow only 2 rests, the rest at A would also have to be eliminated. This would require $d_1 \leq 5.5$ while maintaining $d_1 \geq 9.5$, and is thus infeasible. So $d_1 = 10$ is one optimal solution for a single rest area.

In the previous chapter, we presented Propositions 2.1 and 2.2 which bounded the minimum and maximum number of rests in a cycle, ρ_{min} and ρ_{max}; these bounds in turn lead to bounds on CT_{min} and CT_{max}. Also in that chapter, we demonstrated how these values could be determined by a simple simulation of a driver. As it turns out, the value of ρ_{max} for an optimal configuration of rest facilities can be determined by a simple formula using only the lane length δ_{AB} and the allowable driving hours per day τ_D.

Proposition 3.1. *For any lane length with a minimum feasible number of rest facilities $k = \phi_{min}$, there exists a configuration R' of those k facilities which results in*

$$\rho_{max}(R') = \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil \tag{22}$$

Proof. Let R' be the configuration in which the rest facilities are placed at each multiple of τ_D on the delivery leg. This configuration results in a single cycle time with the number of rests in each cycle given by the the above expression. □

Theorem 3.1. *The value of ρ_{max} for an optimal configuration of any feasible number of rest facilities, denoted ρ^*_{max}, is given by the following expression:*

$$\rho^*_{max} = \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil \tag{23}$$

Proof. From Proposition 3.1, there exists a configuration R' of ϕ_{min} facilities such that:

$$\rho_{max}(R') = \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil \tag{24}$$

For any number of facilities $k \geq \phi_{min}$, an optimal configuration R'' of those k facilities, cannot result in $\rho_{max}(R'') > \rho_{max}(R')$. This is because configuration R' results in the same cycle time for every cycle. If configuration R'' has a higher ρ_{max}, then that configuration cannot be optimal since by Proposition 2.2, $\rho_{max} \leq \rho_{min}+1$, which would imply $\rho_{min}(R'') \geq \rho_{max}(R')$; a better configuration would be to simply use configuration R' with some co-located rest facilities. Thus, $\rho_{max}(R'') \leq \rho_{max}(R')$.

The *minimum* number of rests from Proposition 2.1 is:

$$\rho_{min} \geq \left\lceil \frac{2\delta_{AB}}{\tau_D} - 1 \right\rceil \tag{25}$$

Therefore:

$$\left\lceil \frac{2\delta_{AB}}{\tau_D} - 1 \right\rceil \leq \rho_{min}(R'') \leq \rho_{max}(R'') \leq \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil \tag{26}$$

This range is a single rest, so for any optimal configuration R'' which results in a cycle time sequence which contains cycle times which are not all the same:

$$\left\lceil \frac{2\delta_{AB}}{\tau_D} - 1 \right\rceil = \rho_{min}(R'') < \rho_{max}(R'') = \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil \tag{27}$$

All that is left to show to complete the proof is that for any optimal configuration R''' which results in a cycle time sequence with the same cycle time in every cycle:

$$\left\lceil \frac{2\delta_{AB}}{\tau_D} - 1 \right\rceil < \rho_{min}(R''') = \rho_{max}(R''') = \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil \tag{28}$$

The total distance traveled in n cycles is $2n \cdot \delta_{AB}$. The minimum number of rests needed in order to travel this distance is:

$$\left\lceil \frac{2n \cdot \delta_{AB}}{\tau_D} - 1 \right\rceil = \left\lceil \frac{2n \cdot \delta_{AB}}{\tau_D} \right\rceil - 1 \tag{29}$$

If there exists an $n > 0$ where the total number rests using the lower bound number of rests per cycle is less than the number of rests required to travel n cycles, then our proof will be complete. We must therefore show that there exists an n such that:

$$n \left\lceil \frac{2\delta_{AB}}{\tau_D} - 1 \right\rceil < \left\lceil \frac{2n \cdot \delta_{AB}}{\tau_D} \right\rceil - 1 \tag{30}$$

When $\frac{2\delta_{AB}}{\tau_D} \leq 1$, the left hand side of this equation is always 0 and the inequality will clearly hold for a large enough value of n. When $\frac{2\delta_{AB}}{\tau_D} > 1$, we can let $\frac{2\delta_{AB}}{\tau_D} = i + r$, where i is an integer and $0 < r < 1$. Equation 30 becomes:

$$n \lceil i + r - 1 \rceil < \lceil n(i + r) \rceil - 1 \tag{31}$$

The integer portions can be taken out of the ceiling function:

$$ni - n + n\lceil r \rceil < ni + \lceil nr \rceil - 1 \tag{32}$$

Since $0 < r < 1$, $n\lceil r \rceil = n$ and this equation simplifies to:

$$1 < \lceil nr \rceil \tag{33}$$

Since $0 < r < 1$, it is clear that for a large enough value of n, specifically, for any $n > 1/r$, the right hand side will be greater than 1 and the inequality holds. Therefore the number of rests per cycle must be:

$$\rho_{max}(R''') = \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil \tag{34}$$

Therefore,

$$\rho^*_{max} = \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil \tag{35}$$

□

The significance of this result is that the value of ρ_{max}, and thus CT_{max} for an optimal configuration of rest facilities, depends only on the lane length and is independent of the number of rest facilities, as long as the number of facilities is feasible.

3.2.3 Maximum Number of Cycles

Equation 21 requires that when determining cycle sequence preference, the sum of the cycle times must be compared for all possible values of n which is an infinite number of comparisons. Obviously, an infinite number of comparisons is not feasible within an optimization approach, but fortunately it is also not necessary. So what maximum value of n is sufficient to guarantee identification of a preferred cycle time sequence on a particular lane? Before answering this question, some definitions are necessary.

Consider a sequence of numbers Q of length q. The i^{th} number in sequence Q is denoted q_i.

Definition 3.3. *An* Equivalency Set*, E_i, is the set of indices, j, in sequence Q for which $q_j = q_i$. For example, if $Q = \{10, 20, 10, 20, \ldots\}$ then $E_1 = \{1, 3, 5, \ldots\}$ and $E_2 = \{2, 4, 6, \ldots\}$. We only define the equivalency set for the lowest index in a particular set. In other words, although $E_1 = E_3 = E_5 = \ldots = E_{2k-1}$ for any k, we will only consider these sets as a single equivalency set, E_1.*

Definition 3.4. *Consider 2 sequences of numbers, Q and Q' of length q and q' respectively. Assume $q \geq q'$. Each of these sequences of numbers repeats indefinitely producing an infinite* *sequence of numbers. While the individual numbers in each sequence are not known, assume that it is known that the infinite sequences of numbers produced by the two sequences are identical. Define the* Identifiable Equivalency Set*,* $E^I_i(n)$*, as the set of indices* j *in the range* $1 \leq j \leq q$ *for which it can be shown that* $q_j = q_i$ *by comparing values in the sequences* Q *and* Q' *through the first* n *numbers in each infinite sequence.*

For example, when $q = 6$ and $q' = 2$, the first 7 elements in Q are q_1, q_2, q_3, q_4, q_5, q_6, and q_1. The first 7 elements in Q' are q'_1, q'_2, q'_1, q'_2, q'_1, q'_2, and q'_1.

When $n = 1$, q_1 is compared to q'_1 and the identifiable equivalency sets are: $E^I_1(1) = \{1\}$ and $E^I_2(1) =$ null set. When $n = 2$, q_2 is compared to q'_2 and the identifiable equivalency sets are: $E^I_1(1) = \{1\}$ and $E^I_2(1) = \{2\}$. When $n = 3$, q_3 is compared to q'_1 which has already been shown to be equal to q_1 so the identifiable equivalency sets are: $E^I_1(3) = \{1, 3\}$ and $E^I_2(3) = \{2\}$.

Similarly, for $n = 3, 4, 5, 6$, and 7: $E^I_1(4) = \{1, 3\}$ and $E^I_2(4) = \{2, 4\}$. $E^I_1(5) = \{1, 3, 5\}$ and $E^I_2(5) = \{2, 4\}$. $E^I_1(6) = \{1, 3, 5\}$ and $E^I_2(6) = \{2, 4, 6\}$. $E^I_1(7) = \{1, 3, 5\}$ and $E^I_2(7) = \{2, 4, 6\}$.

Note that for $n \geq 7$, $E^I_j(n) = E^I_j(n-1)$ for all j. Both Q and Q' start again at 1 and the same sequence of comparisons repeats. No further changes to the identifiable equivalency sets can be obtained by comparing the sequences at higher values of n.

In this example, the final number of distinct identifiable equivalency sets was 2, namely, E^I_1 and E^I_2. It is easy to see that Q is partitioned into no more than q' distinct identifiable equivalency sets. The following lemma gives a more precise number of distinct identifiable equivalency sets.

Lemma 3.1. *The final number of distinct identifiable equivalency sets (as* n *approaches infinity) is the highest common factor of* q *and* q'*.*

Proof. Let f be the highest common factor of q and $q\prime$. Then there exists a one to one mapping of sequence Q to sequence R where the i^{th} element of R is a subset of f elements of Q, specifically, $q_{f(i-1)+1}$ to $q_{f(i-1)+f}$. In the previous example, $f = 2$ and $Q = \{q1, q2, q3, q4, q5, q6\}$ maps to $R = \{r1, r2, r3\}$ where $r1 = \{q1, q2\}, r2 = \{q3, q4\}, r3 =$

$\{q5, q6\}$. Similarly, there exists a one to one mapping of sequence Q' to sequence R' where the i^{th} element of R' is a subset of f elements of Q', specifically, $q'_{(f(i-1)+1}$ to $q'_{(f(i-1)+f}$.

Each comparison between elements in R and R' corresponds to a set of f comparisons between elements in Q and Q'. Obviously, the number of identifiable equivalency sets for R is at least 1, and since there is no common factor of r and r', the comparison sequence will not begin to repeat until the $(r * r')^{th}$ comparison and in those $r * r'$ comparisons, every element of R will have been shown to be equivalent to every element of R'. Therefore, the number of distinct identifiable equivalency sets in R is 1 which corresponds to f distinct identifiable equivalency sets in Q. □

The following lemmas follow directly from this result and are presented without proof.

Lemma 3.2. *If f is the greatest common factor of q and q', and q_i and q_j are elements of the same identifiable equivalency set, then $i - j = t \cdot f$ for some integer t.*

Lemma 3.3. *If q_x and q_y are in the same identifiable equivalency set, then q_{x-1} and q_{y-1} are in the same identifiable equivalency set, but not necessarily the same identifiable equivalency set as q_x.*

The following proposition is presented without proof but has been shown to be true by complete enumeration for all combinations of q and $q\prime$ up to $q = 1500$.

Proposition 3.2. *If $E^I_j(n) = E^I_j(n-1)$ for a given n and all j, then $E^I_j(k) = E^I_j(n)$ for all $k > n$ and all j.*

We are now ready to state the key result of this section.

Theorem 3.2. *For a lane with p potential furthest reachable rest facility states, when comparing two rest facility configurations where Equation 21 holds for $n = 1, 2, ..., 2p - 1$ then Equation 21 holds for every $n \geq 2p$.*

Proof. Consider 2 cycle time sequences, C and C'. Let the i^{th} cycle time in sequence C be designated c_i with cycle time t_i and the i^{th} cycle time in sequence C' be designated c'_i with cycle time t'_i. Assume that neither of the two cycle time sequences is preferred to the other. This implies:

$$t_i = t_i' \textit{ for all } i. \tag{36}$$

For cycle time sequence C, let p be the number of distinct driver states and q be the number of those states that repeat. For cycle time sequence C', let p' be the number of distinct driver states and q' be the number of those states that repeat. By convention, $p \geq p'$.

In the initial p' cycles, p' identifiable equivalency sets will each contain a single element. For the next $p - p'$ cycles, an element will be added to an existing equivalency set. For each cycle beyond p, either the comparison will be between two cycles in different equivalency sets in which case a merger of the two equivalency sets will result; or, the two cycles are already in the same equivalency set, in which case, by Proposition 3.2 no further mergers of equivalency sets can be generated by the two cycle sequences.

There are at most p equivalency sets after p cycles. The number of equivalency sets will be reduced by 1 for each cycle beyond p before no mergers are possible. Therefore, a maximum of $p - 1$ cycles beyond p is necessary to guarantee neither cycle time sequence is preferred to the other.

□

The importance of this result is that we only need look at the first $2p - 1$ cycles in determining whether one cycle time sequence is preferred to another.

3.2.4 Additional Modeling Concepts

Some additional concepts used in the models of the following sections are now presented.

Consider a continuous line representing a driver's travel segments from A to B, B to A, A to B, etc. In other words, A is at position 0 on the line, B at position δ_{AB}, A at $2\delta_{AB}$, and so on. In n cycles, there are $2n\delta_{AB}$ driving hours. If a driver rests m times during those n cycles, the maximum distance the driver could travel is $(m + 1)\tau_D$.

Given k rest facilities, the location of the j^{th} facility on the continuous line defined above during the c^{th} cycle is denoted by r_{jc} and is given by the following expression:

$$r_{jc} = 2(c-1)\delta_{AB} + d_j \tag{37}$$

This equation represents the location of the facility on each delivery leg. The position on the return leg of the j^{th} facility on the continuous line defined above on the c^{th} cycle is denoted m_{jc} and is given by the following expression:

$$m_{jc} = 2c\delta_{AB} - d_j. \tag{38}$$

The position of terminal B on the c^{th} cycle is denoted t_{bc} and is given by the following expression:

$$t_{bc} = 2c\delta_{AB} - \delta_{AB}. \tag{39}$$

The position of terminal A on the c^{th} cycle is denoted t_{ac} and is given by the following expression:

$$t_{ac} = 2c\delta_{AB}. \tag{40}$$

3.2.5 MIP Formulation

The concepts presented in the previous sections can now be applied within a mixed integer programming formulation of the optimal location problem for k rest areas. This formulation can be applied to lane lengths $\delta_{AB} \geq \tau_D/2$; note that short-haul lanes are not the primary focus of this research.

3.2.5.1 Variables

The following variables are used in this formulation:

- Decision Variables
  - d_i is the distance of the i^{th} rest facility from A, for $i = 1, 2, ..., k$. These are the primary decision variables for which we are attempting to find optimal values.

- x_i is the total travel distance for a simulated driver from the beginning of the first cycle through the i^{th} required rest.

- Induced Variables

  - c_i is the cycle in which the i^{th} rest occurs.
  - e_i is the distance to the i^{th} overall rest measured from the beginning of the cycle in which the i^{th} rest occurs.
  - s_{ic} is a binary variable which equals 1 when rest i occurs in cycle c.

- Auxiliary Binary Variables

  - y_{ij}, and z_{ij} are auxiliary binary variables which enforce if-then conditions. The y_{ij} variables enforce the condition that if rest i occurs past the distance for rest area location j, then rest i must occur at a distance for rest location $j+1$ or greater. The z_{ij} variables enforce a similar condition for the return leg of the cycle.
  - t_{ic} is an auxiliary binary variable to force the s_{ic} variable to the correct value.

- Input Variables

  - k is the number of rest facilities to be located.
  - ϵ is the minimum allowable separation between rest facilities.
  - δ_{AB} is the lane length.
  - τ_D is the maximum allowable drive time.
  - n_{CYCLES} is the number of cycles to be evaluated. Recall from Theorem 3.2 that for p furthest-rest-area states, $2p-1$ cycles are sufficient to identify a preferred cycle time sequence. Let p be the largest possible number of furthest rest area states: if $\tau_D < \delta_{AB}$, then $p = k$ and $n_{CYCLES} = 2k-1$; if $\tau_D > \delta_{AB}$, then $p = 2k+1$ and $n_{CYCLES} = 4k+1$; if $\tau_D = \delta_{AB}$ then the solution is trivial because no rest facilities are required in the optimal solution.

– n_{RESTS} is the total number of rests to be evaluated and must allow for $\rho^*_{max} = \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil$ rests in each of the cycles to be evaluated, thus $n_{RESTS} = n_{CYCLES} \left\lceil \frac{2\delta_{AB}}{\tau_D} \right\rceil$.

– M is a large integer.

3.2.5.2 Objective Function

In order for this mixed integer program to produce an optimal rest facility configuration, the rest locations must generate the most preferred cycle time sequence. Therefore, the objective function minimizes a weighted sum of the rests per cycle for a certain number of cycles and certain number of rests in a simulated driver cycle sequence. Rests in a given cycle must be weighted high enough so that an additional rest in that cycle results in a higher objective function value than an additional rest in every subsequent cycle. An exponential weighting achieves this goal. The objective function is therefore:

$$min \sum_{c=1}^{n_{CYCLES}} 2^{(n_{CYCLES}-c)} \sum_{i=1}^{(n_{RESTS})} s_{ic} \tag{41}$$

3.2.5.3 Constraints

Sequential rests must either be in the same cycle or separated by a single cycle:

$$c_i - c_{i-1} \leq 1 \qquad i = 1, 2, ..., n_{RESTS} \tag{42}$$

The earlier rest cannot be in a later cycle:

$$c_i - c_{i-1} \geq 0 \qquad i = 1, 2, ..., n_{RESTS} \tag{43}$$

The total distance traveled through the i_{th} rest, x_i, must be within the cycle of the i^{th} rest and at least as far as the first rest facility in that cycle:

$$x_i \leq 2\delta_{AB} c_i \qquad i = 1, 2, ..., n_{RESTS} \tag{44}$$

$$x_i \geq 2\delta_{AB}(c_i - 1) + d_1 \qquad i = 1, 2, ..., n_{RESTS} \tag{45}$$

Travel distance between rests cannot exceed the allowable drive time and cannot be negative:

$$x_i - x_{i-1} \leq \tau_D \qquad i = 1, 2, ..., n_{RESTS} \tag{46}$$

$$x_i - x_{i-1} \geq 0 \qquad i = 1, 2, ..., n_{RESTS} \tag{47}$$

Travel distance between rest facilities cannot exceed the allowable drive distance and must be strictly greater than 0:

$$d_i - d_{i-1} \geq \epsilon \qquad i = 1, 2, ..., k+1 \tag{48}$$

$$d_i - d_{i-1} \leq \tau_D \qquad i = 1, 2, ..., k+1 \tag{49}$$

The following constraint merely assigns the value of e_i:

$$e_i = x_i - 2\delta_{AB}(c_i - 1) \tag{50}$$

The following two constraints enforce the condition that if rest i occurs beyond rest facility j on the delivery leg, then it occurs at least as far as the distance to rest facility $j+1$:

$$d_{j+1} - e_i \leq My_{ij} \qquad i = 1, 2, ..., n_{RESTS}; j = 1, 2, ..., k \tag{51}$$

$$e_i - d_j \leq M(1 - y_{ij}) \qquad i = 1, 2, ..., n_{RESTS}; j = 1, 2, ..., k \tag{52}$$

The following two constraints enforce a similar condition on the return leg:

$$2\delta_{AB} - d_{j-1} - e_i \leq Mz_{ij} \qquad i = 1, 2, ..., n_{RESTS}; j = 1, 2, ..., k+1 \tag{53}$$

$$e_i - 2\delta_{AB} + d_j \leq M(1 - z_{ij}) \qquad i = 1, 2, ..., n_{RESTS}; j = 1, 2, ..., k+1 \tag{54}$$

The following constraints set the indicator variables, s_{ij}. Specifically, they enforce the condition that if rest i occurs in cycle j, meaning $c_i = j$, then $s_{ij} = 1$, otherwise $s_{ij} = 0$.

$$Mt_{ij} + s_{ij} \geq j + 1 - c_i \qquad i = 1, 2, ..., n_{RESTS}; j = 1, 2, ..., n_{CYCLES} + 1 \tag{55}$$

$$M(t_{ij} - 1) - s_{ij} \leq j - 1 - c_i \qquad i = 1, 2, ..., n_{RESTS}; j = 1, 2, ..., n_{CYCLES} + 1 \tag{56}$$

$$\sum_{j=1}^{n_{CYCLES}+1} s_{ij} \leq 1 \qquad i = 1, 2, ..., n_{RESTS} \tag{57}$$

The distance of the 0^{th} rest facility is defined to be 0, and the distance of the $(k+1)^{th}$ rest facility is defined to be the lane length:

$$d_0 = 0 \tag{58}$$

$$d_{k+1} = \delta_{AB} \tag{59}$$

The 0^{th} rest is defined to occur in cycle 0 and the first rest is defined to occur in cycle 1. This constraint limits the applicability of this MIP to lanes longer than $\tau_D/2$:

$$c_0 = 0 \tag{60}$$

$$c_1 = 1 \tag{61}$$

The distance traveled at the 0^{th} rest is defined to be 0:

$$x_0 = 0 \tag{62}$$

$$s_{ij}, t_{ij}, y_{ij}, z_{ij} \quad binary; \quad c_i \quad integer \tag{63}$$

3.2.6 Solving the MIP

The problem was solved using AMPL/CPLEX for lane lengths from 6 to 22 hours and for 1 to 3 rest areas. The shorter lanes were solved for completeness and 22 hours is close to the longest lane in the continental United States. None of the runs took longer than 60 minutes on a standard personal computer and most took less than 10 minutes. A summary of the optimal rest locations and the resulting number of rests per cycle is shown in Table 18. These optimal rest locations are not necessarily unique. Other combinations of rest locations can also generate the same optimal cycle time sequence.

3.2.7 Effects of Poorly Chosen Rest Facility Locations

This mixed integer program was used to determine the rest facility locations used in the computational study in the previous chapter. However, how bad might the results be if

Table 18: MIP Optimal Rest Facility Locations and Resulting Rests Per Cycle for 5-22 Hour Lanes

Lane Length	Rest Fac d_1	Cycle Rests	Rest Fac d_1, d_2	Cycle Rests	Rest Fac d_1, d_2, d_3	Cycle Rests
5.5		(1)		(1)		(1)
6	1	(112)	$1, 2$	(11112)	$1, 4, 5$	(1111112)
7	3	(112)	$0.1, 3.1$	1(112)	$0.1, 0.2, 3.2$	1(112)
8	3	(12)	$0.5, 5.5$	1(12)	$0.1, 0.2, 5.5$	1(12)
9	2	1(2)	$4, 7$	(122)	$0.1, 3.5, 7.5$	12(122)
10	0.1	1(2)	$0.1, 0.2$	1(2)	$0.1, 0.2, 0.3$	1(2)
11	5.5	(2)	$0.1, 5.5$	(2)	$0.1, 0.2, 5.5$	(2)
12	1	(3)	$1, 6.5$	(23)	$4.5, 6.5, 11$	(223)
13	5.5	(3)	$5.5, 11$	(23)	$0.1, 5.5, 10.9$	2(23)
14	3	(3)	$2.5, 8.5$	2(3)	$3, 6, 11$	(233)
15	4	(3)	$0.1, 9.5$	2(3)	$0.1, 0.2, 9.5$	2(3)
16	5	(3)	$0.1, 10.9$	2(3)	$0.1, 0.2, 10.5$	2(3)
16.5	5.5	(3)	$0.1, 5.5$	(3)	$0.1, 0.2, 5.5$	(3)
17	6	(4)	$0.1, 10.9$	3(4)	$5.5, 11, 12$	(34)
18	7	(4)	$4, 7$	3(4)	$1.5, 9.5, 12.5$	(34)
19	8	(4)	$3, 8$	3(4)	$2.5, 8.5, 13.5$	(34)
20	9	(4)	$0.1, 10.9$	3(4)	$0.1, 10.9, 18.1$	3(4)
21	10	(4)	$0.1, 10$	3(4)	$0.1, 10.9, 20.1$	3(4)
22	11	(4)	$0.1, 11$	(4)	$0.1, 0.2, 11$	(4)

Note: Portion of cycle rest sequence in parentheses () repeats.

rest facility locations are chosen poorly? For example, for the 18-hour lane used in the computational study, if rest areas were located at 1 and 12 hours from A, then every driver would be forced to rest at 1, 12, B, 12, and 1 for each and every load. This longer 86-hour cycle time per load will result in more drivers needed to deliver a given load set. The worst possible locations for additional facilities is to add them at places that do not affect the cycle time sequence generated by the rest facilities at 1 and 12, such as anywhere between A and 1 or between 12 and 12.5. These additional rest areas would never be used, making them poor choices. Table 19 shows the effects of poorly chosen rest locations on the number of drivers needed to deliver the same 32 load sets used in the computational study for the 2, 3 and 4 rest facility cases. As indicated by the table, poorly chosen rest locations can have a significant effect on the number of drivers needed.

3.3 Single Lane With No Backhauls-Unrestricted Empty Rest Case

If drivers driving empty trucks are not required to rest at secure rest facilities, Equations (53) and (54) can be eliminated from the MIP as well as all z_{ij} variables. This smaller MIP was solved using AMPL/CPLEX on the same lanes in less than 1 minute on most lane-length/rest-area combinations and in less than 5 minutes in all cases. A summary of the optimal rest locations and the resulting number of rests per cycle is shown in Table 20. Interestingly, improvements in the resulting cycle times occurred on only the 6, 17 and 18-hour lanes.

3.4 The Single Lane With Backhauls Case

Definition 3.5. *A rest area configuration of k rest facilities is said to be* ***single-lane-single-directional optimal with respect to pick-up location T, or*** $1L1OPT(T,k)$*, if the rest area configuration results in the most preferred driver cycle time sequence among all possible configurations of k facilities, when all loads are picked up at T with no backhauls.*

Proposition 3.3. *A $1L1OPT(T,k)$ configuration exists for any $k \geq \frac{\delta_{AB}}{\tau_D} - 1$.*

Definition 3.6. *A rest area configuration of k rest facilities is said to be* ***single-lane-bi-directionally optimal, or*** $1L2OPT(k)$*, if the rest area configuration is both* $1L1OPT(A,k)$

Table 19: Effects of Poorly Chosen Rest Facility Locations on Number of Drivers by Load Set - 18-Hour Lane

Load Set	Drivers Needed Worst Locations	Drivers Needed Best Locations 2RF	3RF	4RF	Percent Increase in Drivers
$U0$	11	10	10	9	$10 - 22\%$
$U1$	12	10	10	10	20%
$U2$	12	10	10	10	20%
$U3$	12	10	10	10	20%
$U4$	12	10	10	10	20%
$U5$	12	10	10	10	20%
$U6$	16	13	13	13	23%
$U7$	16	14	14	14	14%
$U8$	15	14	12	12	$7 - 25\%$
$U9$	17	13	13	13	31%
$U10$	15	12	12	12	25%
$LU0$	31	27	26	26	$15 - 19\%$
$LU1$	33	28	27	26	$18 - 27\%$
$LU2$	33	28	27	27	$18 - 22\%$
$LU3$	32	27	27	26	$19 - 23\%$
$LU4$	33	28	27	26	$18 - 27\%$
$LU5$	33	28	27	26	$18 - 27\%$
$LU6$	38	30	30	30	27%
$LU7$	37	29	29	29	28%
$LU8$	37	30	30	30	23%
$LU9$	36	29	28	28	$24 - 29\%$
$LU10$	38	31	31	30	$23 - 27\%$
$E1$	21	20	20	20	5%
$E2$	20	15	15	15	33%
$E3$	22	19	19	19	16%
$E4$	24	20	20	20	20%
$E5$	20	18	18	18	11%
$LE1$	47	40	40	40	17.5%
$LE2$	44	35	35	35	26%
$LE3$	48	39	39	39	23%
$LE4$	44	37	37	37	19%
$LE5$	45	38	38	38	18%

Table 20: MIP Optimal Rest Facility Locations and Resulting Rests Per Cycle When Empty Truck Rest is Unrestricted for 5.5-22 Hour Lane Lengths

Lane Length	Rest Fac d_1	Cycle Rests	Rest Fac d_1, d_2	Cycle Rests	Rest Fac d_1, d_2, d_3	Cycle Rests
5.5		(1)		(1)		(1)
6	**1**	**(1111112)**	**0.1,5**	**1(1111112)**	**3,4,5**	**(111111112)**
7	2	(112)	2, 5	1(112)	2, 4, 5	1(112)
8	6	(12)	0.1, 6	1(12)	0.1, 0.2, 6	1(12)
9	0.1	1(2)	4, 8	(122)	0.1, 4, 8	12(122)
10	2	1(2)	0.1, 2	1(2)	0.1, 2, 4	1(2)
11	5.5	(2)	0.1, 5.5	(2)	0.1, 0.2, 5.5	(2)
12	1	(3)	1, 10	(23)	1, 3, 10	(223)
13	2	(3)	2, 9	(23)	3, 7, 11	2(23)
14	3	(3)	3, 11	2(3)	5, 10, 11	(233)
15	4	(3)	3, 11	2(3)	0.1, 0.2, 11	2(3)
16	5	(3)	0.1, 11	2(3)	0.1, 0.2, 11	2(3)
16.5	5.5	(3)	0.1, 5.5	(3)	0.1, 0.2, 5.5	(3)
17	6	(4)	**6,11**	**3(4)**	**9,10,11**	**(334)**
18	7	(4)	**8,11**	**3(4)**	**0.1,8,11**	**3(34)**
19	11	(4)	0.1, 11	3(4)	6, 11, 16	(34)
20	11	(4)	0.1, 11	3(4)	0.1, 0.2, 11	3(4)
21	11	(4)	0.1, 11	3(4)	0.1, 0.2, 10	3(4)
22	11	(4)	0.1, 11	(4)	0.1, 0.2, 11	(4)

Note: **Bold** entries are improvements from the restricted empty rest case.

and $1L1OPT(B,k)$, where A and B are the lane terminals.

Proposition 3.4. *$1L2OPT(k)$ rest area configurations may not exist for a given k.*

Proof. Consider the 10-hour lane case with k=1. Only rest locations between 0 and 1 are $1L1OPT(A,1)$ locations, and only rest locations between 9 and 10 are $1L1OPT(B,1)$. Since these ranges are mutually exclusive, no $1L2OPT(1)$ configuration exists. □

For the single lane with backhauls problem, in addition to cycle times, we must also take into consideration single leg times.

Definition 3.7. ***Minimum Leg time***, *or LT_{min}, is the minimum time it takes a fully rested driver to transit a lane in one direction and be ready to start a load at the other terminal and is given by the following expression: $LT_{min} = \delta_{AB} + \tau_R * \rho^1_{min}$.*

Definition 3.8. ***Maximum Leg time***, *or LT_{max}, is the maximum required time for any driver to transit a lane in one direction and be ready to start a load at the other terminal and is given by the following expression: $LT_{max} = \delta_{AB} + \tau_R * \rho^1_{max}$.*

A given rest area configuration will result in a specific leg time sequence when starting at A, and possibly a different leg time sequence when starting at B. In order to compare rest area configurations, a definition is needed that allows the determination of a preference order on leg time sequences, similar to the preference between cycle time sequences.

Definition 3.9. *Let $LT_{i\alpha}$ be the leg time of the i^{th} leg in the leg time sequence α. Then a leg time sequence α is* ***preferred*** *to leg time sequence β if and only if:*

$$\sum_{i=1}^{n} LT_{i\alpha} \leq \sum_{i=1}^{n} LT_{i\beta} \tag{64}$$

for all n, and strict preference requires that the expression holds with strict inequality for at least one value of n.

Definition 3.10. *A rest area configuration of k rest facilities is said to be* ***single lane leg optimal with respect to terminal T***, *or $OPT1(T,k)$, if the rest area configuration results in the most preferred leg time sequences when starting from terminal T.*

Definition 3.11. *A rest area configuration of k rest facilities is said to be **single lane optimal**, or $OPT(k)$, if the configuration is both $OPT1(A, k)$ and $OPT1(B, k)$.*

Proposition 3.5. *If rest area configuration R is $OPT(k)$, then R is also $1L2OPT(k)$.*

Proof. This is clearly the case since each cycle time in a cycle time sequence is simply the sum of the 2 leg times in a leg time sequence. If each leg time sequence is the most preferred, the associated cycle time sequences must also be the most preferred. □

Proposition 3.6. *If rest area configuration R is $1L2OPT(k)$, then R is not necessarily $OPT(k)$.*

Proof. Consider an 18-hour lane with 3 rest areas at 1.5, 9.5, and 12.5 hours from A. This configuration is $1L2OPT(3)$ resulting in a cycle time sequence of 66 and 76 hours with both states repeating indefinitely for both the A to B and the B to A directions. This configuration is not $OPT(3)$, however, because the first leg time in the A to B direction is 38 hours while the first leg time in the B to A direction is 28 hours.

□

Unlike the single lane with no back hauls case, when backhaul loads must be considered, optimal rest locations can not generally be determined independent of the load set. There are conditions, however, when they can be determined independent of the load set and we develop those conditions now.

Theorem 3.3. *If an $OPT(k)$ configuration exists for a given lane and value of k, that configuration is optimal for any possible load set for the single lane problem with backhauls.*

Proof. Let $R1$ be an $OPT(k)$ rest area configuration. Assume $R1$ is not the optimal configuration for load set L'. This implies another rest area configuration, $R2$, results in fewer drivers required to deliver L'. Let $X^i \subseteq L'$ be the set of loads assigned to driver i in the resulting optimal driver to load assignment under $R2$. Then there must exist an X^i which can not be feasibly delivered by a driver under rest area configuration $R1$. However, $R1$ being $OPT(k)$ means that for any n legs, the sum of the leg times under $R1$ is less than or equal to the sum of leg times under any other configuration, including $R2$, so X^i can be delivered by a driver under rest area configuration $R1$ which is a contradiction.

□

Theorem 3.4. *If an $OPT(k)$ configuration does not exist for a given lane and value of k, then the optimal configuration of those k rest facilities depends on the load set.*

Proof. We provide a counterexample. Consider the same 10-hour lane discussed above with $k = 1$. Only rest locations between 0 and 1 are $1L1OPT(A, 1)$ locations, and only rest locations between 9 and 10 are $1L1OPT(B, 1)$. Since these ranges are mutually exclusive, no $1L2OPT(1)$ configuration exists and no $OPT(1)$ configuration exists. Further, the $1L1OPT(A, 1)$ optimal location results in a cycle time sequence for loads originating at A of 30 hours on the first cycle and 40 hours on every subsequent cycle, and a cycle time sequence for loads originating at B of 40 hours for every cycle. The $1L1OPT(B, 1)$ optimal location results in the same cycle times but for the opposite terminals. Now consider the following 2 load sets. L_1 has all load pickups at terminal A with pickup times at 0, 30, 70, and 110. L_2 has all load pickups at terminal B with pickup times at 0, 30, 70, and 110. Any $1L1OPT(A, 1)$ location will result in 1 driver needed to deliver L_1 and 2 drivers to deliver L_2. Any $1L1OPT(B, 1)$ location will result in 2 drivers needed to deliver L_1 and 1 driver to deliver L_2. Therefore, the preferred configuration depends on the load set. □

In Table 18, under the single rest area column, each of the indicated rest area locations results in the same optimal driver cycle time sequence whether drivers begin at A or B, except for lane lengths in the range: $8.25 \leq \delta_{AB} < 11$. However, the optimal rest locations listed in Table 18 are not unique. Figure 11 shows the entire range of single lane one direction optimal ($1L1OPT$) rest locations for a single rest area and also the ranges of locations that are single lane two direction optimal ($1L2OPT$).

Clearly, the set of rest locations that are $1L2OPT$ is a subset of the set of rest locations that are $1L1OPT$. The symmetry becomes apparent when the x-axis is changed to percent of the lane length as in Figures 12 and 13. One interesting aspect of these figures is that for most lane lengths using a single rest facility, all $1L1OPT$ rest locations are also

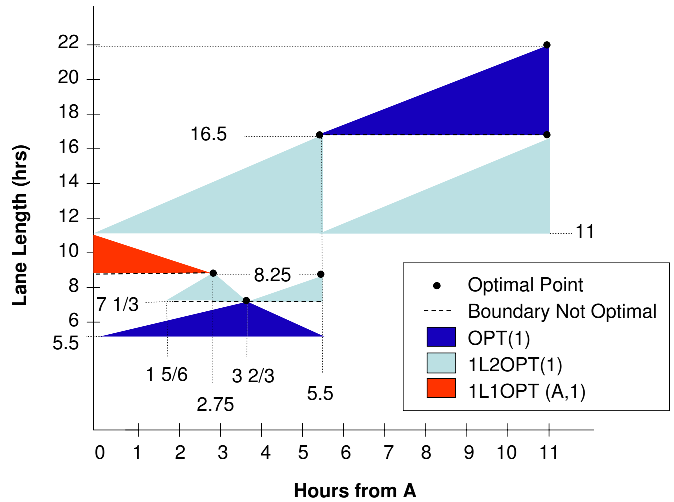

Figure 11: Single Lane Single Source Optimal Rest Locations (in Hours From Source) for One Rest Area for 5.5-22 Hour Lane Lengths

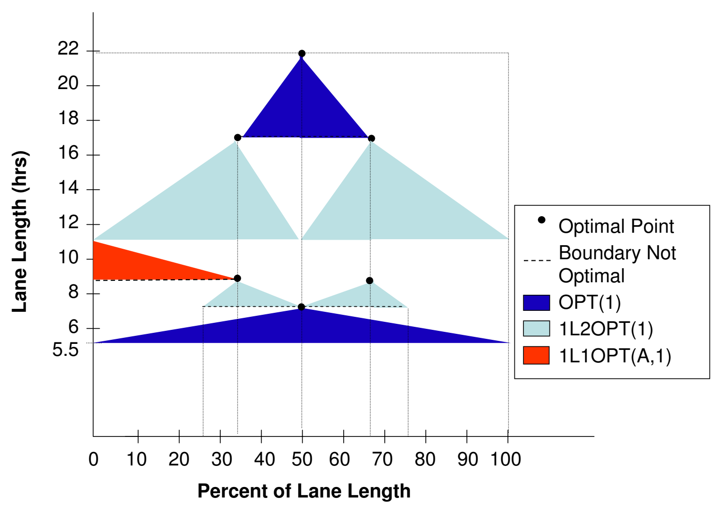

Figure 12: Single Lane Single Source Optimal Rest Locations (in Percent of Lane Length from Source) for One Rest Area for 5.5-22 Hour Lane Lengths

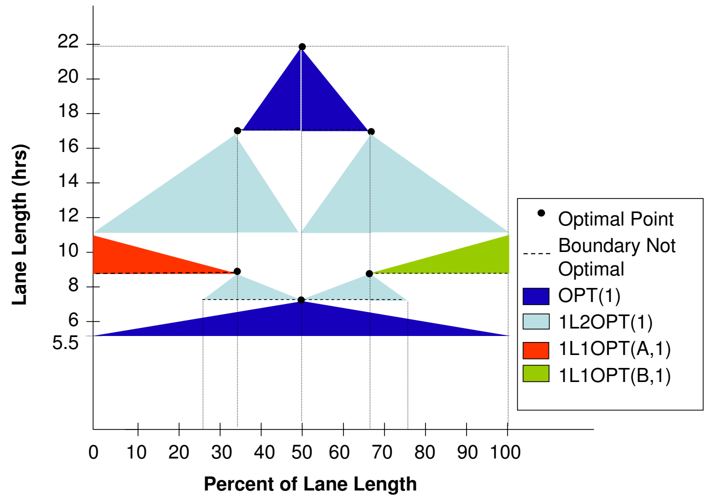

Figure 13: Single Lane Single Source Optimal Rest Locations (in percent of Lane Length from A) for One Rest Area for 5.5-22 Hour Lane Lengths With Either A or B as the Source

$1L2OPT$. This does not necessarily extend to larger numbers of rest facilities. Also the set of rest locations that are $OPT(k)$ is a subset of the set of rest locations that are $1L2OPT$. $OPT(k)$ rest locations require the locations to be symmetric in the sense that they must allow identical leg cycles in either direction. Figure 14 illustrates the relationships between the different types of optimality for rest area configurations.

A closer look at the structure of these figures is warranted. Consider Figure 13.

- *For lane lengths* $16.5 < \delta_{AB} \leq 22$: The areas to the left and right of the highlighted optimal locations are not only not optimal, but they are also infeasible because they are more than 11 hours from either terminal A or B. Furthermore, each feasible location is not close enough to either terminal to allow a driver to cycle through the terminal without resting. For every load, the driver will rest at both terminals and at the rest facility in each direction whether starting from A or B. For these lane lengths, all feasible rest facility locations are $OPT(1)$.

- *For lane lengths* $11 < \delta_{AB} \leq 16.5$: There are two distinct optimal regions that are $1L2OPT(1)$. The interval between these two optimal regions is feasible, but suboptimal. If the facility were located in this suboptimal region, it would not be close

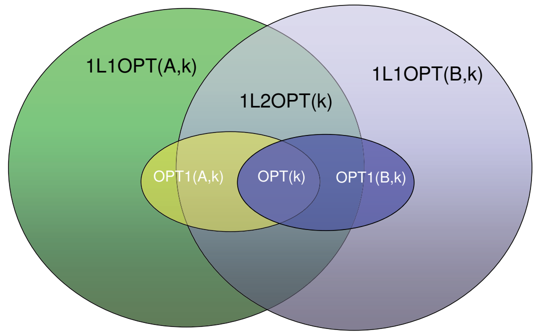

Figure 14: Relationships Between Different Types of Optimality for Rest Area Configurations

enough to either terminal to allow a driver to cycle through the terminal without resting, and the same cycle time sequence as that on the longer 1.5 to 22 hour lane would result. Alternatively, facilities located in the highlighted optimal region allow a driver who rests at that facility to cycle through the closest terminal and return to that rest facility for the subsequent rest. This results in 3 rests per cycle for drivers starting from either terminal. These locations are only $1L2OPT(1)$ and not $OPT(1)$ because the number of rests in the first leg depends on for the terminal from which the driver is dispatched. For example, for the 16.5 hour lane with the rest facility at 5.5, a driver starting at A must rest at the rest facility and at B on the first leg for a total of 2 rests on the first leg, but a driver starting at B only must rest at the rest facility for a total of 1 rest on the first leg. The results when the rest facility is located at 11 are similar due to the symmetry of the problem. The interval on the figure from 0 to the optimal region, and the interval greater than 11 are both infeasible because facilities in those intervals would lie more than 11 hours from one of the terminals.

- *For lane lengths* $\delta_{AB} = 11$: No rest facilities are needed because drivers must always rest at both A and B.

- *For lane lengths* $8.25 < \delta_{AB} < 11$: All possible rest locations are feasible, but none of the locations outside the shaded optimal regions would ever be used by a driver. A driver using a facility in such a location cannot have sufficient drive hours to cycle through the terminal and reach them again on the reverse leg, or would always have sufficient remaining drive hours to skip them and reach the next terminal. For these non-optimal locations, drivers must always rest at each terminal. The highlighted optimal locations allow a driver who rests at the furthest terminal from the rest facility to cycle through the opposite terminal and reach this rest facility on the next driving day. For loads starting at the terminal nearest the rest facility, a driver would rest at the opposite terminal on the first leg and not have to rest at the origin terminal on the following leg. However, every subsequent cycle would result in a rest at the rest facility and at the destination terminal. For loads originating at the terminal furthest from the rest facility, the driver must rest at the facility and at the origin terminal for every load. This results in different cycle time sequences depending on which terminal is the origin and only one of the cycle time sequences is optimal. Therefore, the locations highlighted in the figure are $1L1OPT$ for the terminal nearest the rest location.

- *For lane lengths* $7\frac{1}{3} < \delta_{AB} \leq 8.25$: All possible rest locations are feasible. The optimal intervals represent facility locations which meet two conditions. First, a facility in this interval can be reached in a single driving day by a driver starting at the furthest terminal who drives to the nearer terminal and stops at the rest facility on the return leg. Second, on the subsequent driving day, the driver can cycle through the further terminal and reach the rest facility which allows the driver to reach the origin terminal on the third driving day. In other words, the location is within $\tau_D/2$ of one terminal and within $\tau_D - \delta_{AB}$ of the other terminal. The resulting cycle time sequence is the same whether a driver starts at terminal A or B. For example, for the 8-hour lane with the rest facility 3 hours from A, for loads originating at A, the first cycle would include a single rest at B. The subsequent cycle will include rests at the facility on the delivery leg and on the return leg. This cycle sequence will then repeat. For loads originating at B, the first cycle will include a single rest at the facility on the return leg. The second cycle will include a rest at the facility on the delivery leg and at B. This cycle sequence will then repeat. These locations are not $OPT(1)$ because the first leg time will either contain 1 or 2 rests depending upon the originating terminal. For the two $1L2OPT(1)$ intervals in the figure, the leftmost interval will result in loads originating at terminal B having the preferred leg time sequence while the rightmost interval will result in loads originating at terminal A having the preferred leg time sequence. There are three regions of suboptimal locations for these lane lengths. The interval between the two optimal intervals represents those locations that would result in drivers resting only at A and B with 2 rests per cycle for every cycle. Note that this is the same sequence of rests that results for locations within the contiguous suboptimal region for the 8.25 to 11 hour lane lengths. The leftmost and rightmost suboptimal regions represent locations that allow a driver to reach the rest facility when starting from the distant terminal and after cycling through the nearer terminal, but do not allow the driver to reach the facility on the subsequent leg after passing through the distant terminal. In other words, the rest facility is located within $\tau_D - \delta_{AB}$ of one terminal, but not within $\tau_D/2$ of the other terminal, resulting in two rests per cycle for every cycle when starting from the furthest terminal from the rest facility, and one rest on the first cycle and two rests on every subsequent cycle when starting from the terminal nearest the rest facility.

- *For lane lengths* $5.5 < \delta_{AB} \leq 7\frac{1}{3}$: All rest locations are feasible. The optimal interval represents those facility locations which allow a driver to reach the rest facility in a single driving day when starting from either terminal after cycling through the opposite terminal. In other words, the location is within $\tau_D - \delta_{AB}$ of each terminal. This allows a driver starting at a terminal to rest at the rest facility on the return leg of his first cycle, at the opposite terminal on his second cycle, and at the rest facility and at the end of his third cycle. This results in identical leg time sequences and cycle time sequences from both A and B and these facility locations are therefore $OPT(1)$. The suboptimal intervals to the left and right of the optimal region represent locations where the facility is located within $\tau_D - \delta_{AB}$ of only one of the terminals. This results in two rests per cycle for loads originating at the more distant terminal and a cycle rest sequence of one rest on the first cycle and two rests on every subsequent cycle for loads originating at the terminal nearest to the rest facility.

Clear graphical representations of alternative optimal rest facility locations using a two dimensional figure for multiple rest areas are difficult to create. For two rest facilities, the intervals containing alternative optimal locations for the two facilities are interdependent. For example, on the 18-hour lane with 2 rest facilities, the MIP optimal configuration is $d_1 = 4$ and $d_2 = 7$. This results in the driver resting at r_2, B, and r_2 on the first cycle, and at r_1, r_2, B, and r_2 on every subsequent cycle. The characteristics which enable this cycle time sequence are: r_2 is within 11 hours of both A and B, and rest at A is not required because $d_1 + d_2 \leq 11$. The combinations of d_1 and d_2 which result in this cycle sequence are displayed graphically in Figure 15.

The first step to identify alternative optimal rest facility locations is to determine the MIP solution for optimal rest facility locations. A driver is then simulated through the cycle time sequence, and a linear constraint for each driving leg is generated that ensures that the rest facility at the start of the leg and the rest facility at the end of the leg are separated by no more than τ_D driving hours. Algorithm 1 formalizes this procedure. Note that this algorithm defines distances for the rests on the return leg so that terminal B is considered the $(k+1)^{st}$ rest location, r_k on the return leg is the $(k+2)^{nd}$ rest location, and so forth.

As an example, consider a problem where $\delta_{AB} = 18$, $k = 2$, and a MIP solution of $d_1 = 4$ and $d_2 = 7$, with $\tau_D = 11$. The algorithm would proceed as follows:

- Input $k = 2$; $\delta_{AB} = 18$; $\tau_D = 11$; $d_1 = 4$; $d_2 = 7$.
- Define $d_0 = 0$; $d_3 = 18$; $d_4 = 29$; $d_5 = 32$.
- Initialize $X = \{\}$; $S = \{\}$; $x = 0$.
- Since $36 - 0 > 11$, set $y = 2$.

Algorithm 1: Identifying Alternate Optimal Rest Facility Locations

Inputs:

k, the number of rest facilities.

δ_{AB}, the lane length.

τ_D, allowable single-day drive time.

$d(R) = \{d_1, d_2, \cdots, d_k\}$, the set of optimal rest locations in optimal rest configuration R.

Output:

S, a set of constraints defining a region of alternate optimal rest facility locations.

Main:

DEFINE $d_0 = 0$, rest at terminal A
DEFINE $d_{k+1} = \delta_{AB}$, rest at terminal B
DEFINE $d_{k+1+j} = 2\delta_{AB} - d_{k-j+1}$ for $j = 1, 2, ..., k$, rest on return leg
INITIALIZE $X = \{\}$, set of drive start points already evaluated
INITIALIZE $S = \{\}$, start with empty set of constraints
INITIALIZE $x = 0$, initial driver location at terminal A
LOOP WHILE $x \notin X$:
  IF $2\delta_{AB} - d_x > \tau_D$ (rests in same cycle):
    SET $y = i$ where i is the largest i that satisfies $d_i - d_x \leq \tau_D$
  ELSE IF $x \geq k + 1$ (x is on return lane, leg will pass through terminal A):
    SET $y = i$ where i is the largest i that satisfies $d_i + d_x - 2\delta_{AB} \leq \tau_D$
  ELSE (x is on delivery lane, leg will pass through B and A):
    SET $y = i$ where i is the largest i that satisfies $d_i + \delta_{AB} + \delta_{AB} - d_x \leq \tau_D$
  END IF   EvaluateLeg(x,y)
  ADD x to set X
  SET $x = y$
END LOOP

EvaluateLeg(r1,r2):

IF $2\delta_{AB} - d_{r_1} > \tau_D$ (r_1 and r_2 are in the same cycle):
  CASE $r_2 \leq k + 1$ (both rests on delivery leg)
    SET $s = \{d_{r_2} - d_{r_1} \leq \tau_D\}$
  CASE ($r_2 > k + 1$ AND $r_1 \leq k + 1$)
    SET $s = \{\delta_{AB} - d_{r_1} + \delta_{AB} - d_{2k+2-r_2} \leq \tau_D\}$
  CASE ($r_2 > k + 1$ AND $r_1 > k + 1$)
    SET $s =\}d_{2k+2-r_1} - d_{2k+2-r_2} \leq \tau_D\}$
  END CASE
ELSE IF $r_1 \geq k + 1$ (r_1 is on return lane, leg will pass through terminal A):
  CASE ($r_2 \leq k + 1$):
    SET $s = \{d_{r_2} + d_{2k+2-r_1} \leq \tau_D\}$
  CASE ($r_2 > k + 1$) (leg passes through terminals A and B):
    SET $s = \{d_{2k+2-r_1} + \delta_{AB} + \delta AB - d_{2k+2-r_2} \leq \tau_D\}$
  END CASE
ELSE(r_1 is on delivery lane, leg will pass through B and A):
  SET $s = \{d_{r_2} + \delta_{AB} + \delta_{AB} - d_{r_1} \leq \tau_D$
END IF
ADD constraint s to set S
RETURN

- Evaluate Leg (0,2) results in adding the constraint $\{d_2 \leq 11\}$ to set S.
- 0 is added to set X. $X = \{0\}$.
- Set $x = 2$.
- Since $36 - 7 > 11$, set $y = 3$.
- Evaluate Leg (2,3) results in adding the constraint $\{18 - d_2 \leq 11\}$ to set S.
- 2 is added to set X. $X = \{0, 2\}$.
- Set $x = 3$.
- Since $36 - 18 > 11$, set $y = 4$.
- Evaluate Leg (3,4) results in adding the constraint $\{18 - d_2 \leq 11\}$ to set S.
- 3 is added to set X. $X = \{0, 2, 3\}$.
- Set $x = 4$.
- Since $36 - 29 > 11$ is FALSE, and $x \geq k + 1$, set $y = 1$.
- Evaluate Leg (4,1) results in adding the constraint $\{d_2 + d_1 \leq 11\}$ to set S.
- 4 is added to set X. $X = \{0, 2, 3, 4\}$.
- Set $x = 1$
- Since $36 - 4 > 11$, set $y = 2$.
- Evaluate Leg (1,2) results in adding the constraint $\{d_2 - d_1 \leq 11\}$ to set S.
- 1 is added to set X. $X = \{0, 1, 2, 3, 4\}$.
- Set $x = 2$
- Since $x = 2 \in X$, the algorithm terminates.

After eliminating redundant constraints from S, the resulting set of linear inequalities are:

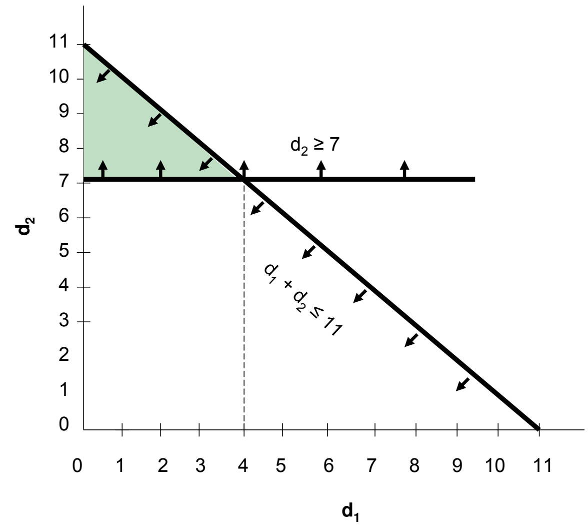

Figure 15: Alternate Single Lane Single Source Optimal Rest Facility Locations for 2 Facilities on an 18-Hour Lane

- $d_2 \geq 7$.
- $d_1 + d_2 \leq 11$.

These, along with the non-negativity constraints, are the equations graphed in Figure 15 showing the range of solutions which provide the same cycle time sequence as the MIP solution.

In some cases, the rest areas listed in Table 18 are $1L1OPT$ but not $1L2OPT$, but there exist alternate locations that are $1L2OPT$, or possibly even $OPT(k)$. Of course, since the MIP is designed to find only $1L1OPT$ solutions, this is not necessarily unexpected. An example of this can be shown for the 10-hour lane. The indicated optimal locations of 0.1 and 0.2 lead to the following sequences of required rests:

- For drivers starting at A: one rest on the first cycle followed by two rests on every subsequent cycle.
- For drivers starting at B: two rests on every cycle.

Locating the rests at 1 and 9, however, would result in all drivers having one rest on the first cycle followed by two rests on every subsequent cycle, irrespective of the initial terminal. It should also be clear that since this configuration is perfectly symmetric, the configuration is also $OPT(2)$.

Finding $1L2OPT(k)$ configurations is possible using a similar MIP. In this case, the MIP will attempt to minimize a weighted sum of rests per cycle for two different cycle sequences. The first is the sequence modeled in the existing MIP. The second is a cycle sequence starting from the opposite terminal. Additional auxiliary variables and constraints would be necessary, effectively doubling the size of the problem, with an additional constraint for each rest facility which forces the same location for respective rest facilities in the two cycle sequences. In other words, two distinct location problems are solved simultaneously with distinct location decision variables, d_i^A and d_i^B, with additional constraints that require $d_1^A = d_k^B$, $d_2^A = d_{k-1}^B$, and so forth.

This modified formulation will find a configuration which gives the best possible combined cycle time sequence. Obviously, if a $1L2OPT(k)$ solution exists, this modified MIP will find it. But if there is no $1L2OPT(k)$ solution, the MIP will still find the configuration which minimizes the sum of the combined weighted averages of the rests per cycle which may not necessarily be a $1L1OPT$ solution for either the A to B lane or the B to A lane. To illustrate how this might occur, consider the case where the $1L1OPT$ solution results in a rests per cycle sequence of (12), where the portion in () repeats, and the resulting rests per cycle starting from the other terminal for this configuration is (2). If a configuration exists in which the rests per cycle from either terminal is 1(2), this configuration would be the MIP result, because the first cycle from either terminal A or terminal B would require only one rest, while the $1L1OPT$ solution requires one rest on the first cycle from terminal A, but two rests on the first cycle from terminal B.

Evaluation of the solution generated from this modified MIP is necessary to determine whether it is $1L1OPT$ or $1L2OPT$, but this is easily accomplished using a simple forward simulation on the generated solution from each terminal, and comparing the cycle time sequences to the single lane without backhaul optimal cycle time sequence from the original MIP such as in Table 18.

Finding $OPT(k)$ configurations is also possible by further modifying the MIP. In this case, leg time sequences would need to be evaluated instead of cycle time sequences. This would in effect double the number of cycles (legs) to be evaluated, but not change the total number of rests to be evaluated within the formulation. Cycles beginning from each terminal would need to be evaluated, with the same considerations and effects discussed above in the MIP modifications for finding $1L2OPT(k)$ configurations. Tractability and implementation of these modified MIP's is left for future study.

Table 21 lists some $1L2OPT$ and $OPT(k)$ locations. The $1L2OPT$ configurations were determined by starting with the Table 18 $1L1OPT$ solution, and identifying the alternative optimal locations using Algorithm 1. The mirror image of this region of optimal solutions was then identified. These mirror image solutions correspond to optimal rest facility locations when dispatching from terminal B instead of from terminal A. The intersection of the optimal region and the mirror image optimal region correspond to $1L2OPT$ solutions. In the single rest facility case, since the optimal interval for the rest facility is a unique interval, it is easy to show that if there is no intersection of the optimal region and its mirror image, then no $1l2OPT$ configuration exists. For two or more rest facilities, the Algorithm 1 regions are not guaranteed to be the complete set of alternate optimal solutions. To determine if the solution was $OPT(k)$, simulations of a driver dispatched from terminal A and one dispatched from terminal B using the $1l2OPT$ configuration are compared for identical leg time sequences. If the leg time sequences are identical, the configuration is $OPT(k)$.

3.4.1 Myopic Rest Facility Location Algorithm

As demonstrated in the previous section, for a single lane with backhauls, when $OPT(k)$ configurations do not exist optimal rest facility locations depend upon the load set. This section presents a myopic strategy for locating rest facilities on a lane for a specified load set, and bounds the worst case performance of this strategy.

The basic procedure is to use an $OPT(k)$ configuration if one exists, otherwise to use a $1L2OPT$ configuration if it exists, otherwise to use a $1L1OPT$ configuration. If the

Table 21: Some Single Lane Bi-Directionally Optimal ($1L2OPT$) Rest Facility Configurations for 6-22 Hour Lanes with 1-4 Rest Facilities

Lane Length	1 RF d_1	2 RF d_1, d_2	3 RF d_1, d_2, d_3	4 RF d_1, d_2, d_3, d_4
6.0	1^*	$1, 2$	$1, 2, 5$	$1, 2, 3, 4$
7.0	3^*		$0.1, 3.5, 6.9^*$	$1.5, 3, 4.5, 6.5$
8.0	3			$0.5, 2.5, 5.5, 7.5^*$
10	NE	$1, 9^*$	$1, 9^*$	$1, 9^*$
11	**	**	**	**
12	1.0	$1, 6.5$		
13	5.5	$5.5, 11$		
14	3		$5, 6, 11$	
15	4			
16.5	5.5	$5.5, 11^*$	$5.5, 11^*$	$5.5, 11^*$
18	7^*		$1.5, 9.5, 12.5$	$3, 8, 11, 14^*$
20	9^*		$1, 10, 19^*$	$1, 10, 19^*$
21	10^*		$0.5, 10.5, 20.5^*$	$0.5, 10.5, 20.5^*$
22	11^*	11^*	11^*	11^*

*The configuration is Single Lane Optimal, $OPT(k)$.
**No rest facilities are needed.
NE = No $1L2OPT$ configuration exists.
Blank entries represent configurations which were not analyzed.

configuration is $OPT(k)$, it is optimal for any load set. Alternatively, if the best available configuration is either $1L1OPT$ or $1L2OPT$, Algorithm 2 will identify the preferred orientation of the configuration based on the load set.

Definition 3.12. *A rest configuration, R^M, is said to be a* ***mirror image configuration of*** *R if and only if there is a one-to-one mapping where each rest facility, $r \in R$, corresponds to a rest facility, $r^M \in R^M$, such that the distance of r from terminal A equals the distance of r^M from terminal B and the number of rest facilities in R equals the number of rest facilities in R^M.*

Algorithm 2 runs in polynomial time and selects the preferred orientation by using the one with the lowest maximum multi-cycle lower bound. Note that determining the initial configuration is not polynomial, but Tables 18 and 21 contain $1L1OPT$ and $1L2OPT$ solutions for several lane lengths.

It is easy to show that this myopic algorithm will produce solutions requiring no more than four times the number of drivers required in the optimal configuration.

Algorithm 2: Single Lane with Backhauls Load-Dependent Rest Facility Location Algorithm

Inputs:

R, a $1L1OPT$ or $1L2OPT$ rest facility configuration.

L^A, the set of load pickup times for loads to be picked up at terminal A

L^B, the set of load pickup times for loads to be picked up at terminal B

Output:

R^*, Chosen rest facility configuration.

Main:

SET R^M = the mirror image configuration of R

SET α_A = the resulting cycle time sequence starting at terminal A for configuration R

SET α_B = the resulting cycle time sequence starting at terminal B for configuration R

SET $\alpha_A^M = \alpha_B$, the resulting cycle time sequence starting at terminal A for configuration R^M

SET $\alpha_B^M = \alpha_A$, the resulting cycle time sequence starting at terminal B for configuration R^M

CALCULATE Λ_A = the multi-cycle lower bound for L_A using cycle time sequence α_A

CALCULATE Λ_B = the multi-cycle lower bound for L_B using cycle time sequence α_B

CALCULATE Λ_A^M = the multi-cycle lower bound for L_A using cycle time sequence α_A^M

CALCULATE Λ_B^M = the multi-cycle lower bound for L_B using cycle time sequence α_B^M

SET $t = \max(\Lambda_A, \Lambda_B)$

SET $t^M = \max(\Lambda_A^M, \Lambda_B^M)$

IF $t^M < t$:

  SET $R^* = R^M$

ELSE:

  SET $R^* = R$

END IF

EXIT

Proposition 3.7. *If the maximum number of rests required for a $1L1OPT$ configuration R is z, then the maximum number of rests required for its mirror configuration R^M is also z:*

$$\rho_{max}(R) = \rho_{max}(R^M) \tag{65}$$

Proof. Assume terminal A is the origin terminal and B is the destination terminal for the $1L1OPT$ configuration. Clearly $\rho_{max}(R^M)$ cannot be less than $\rho_{max}(R)$ since R is $1L1OPT$. $\rho_{max}(R^M)$ also cannot be more than $\rho_{max}(R)$ since a driver dispatched from terminal B could utilize the same sequence of rest facilities utilized by the driver starting at terminal A, and therefore generate an identical leg time sequence.

□

Theorem 3.5. *Algorithm 3 will produce a configuration requiring no more than four times the optimal number of drivers on a single lane with backhauls.*

Proof. By Corollary 2.1.1, the maximum cycle time for any optimal configuration of rest facilities depends only on the lane length. By Proposition 3.7 the maximum number of rests, and therefore the maximum cycle time for the mirror configuration is the same as for the original configuration. Therefore, $\Omega_A = \Omega_A^M$ and $\Omega_B = \Omega_B^M$. It is therefore easy to see that any chosen configuration will result in at most $\Omega_A + \Omega_B$ drivers required to deliver the loads.

If $\hat{z}$ is the number of drivers required for a chosen configuration, clearly

$$\frac{\hat{z}}{z^*} \leq \frac{\Omega_A + \Omega_B}{z^*} \tag{66}$$

Since $z^* \geq \min[\max(\Lambda_A, \Lambda_B), \max(\Lambda_A^M, \Lambda_B^M)]$:

$$\frac{\hat{z}}{z^*} \leq \frac{\Omega_A + \Omega_B}{\min[\max(\Lambda_A, \Lambda_B), \max(\Lambda_A^M, \Lambda_B^M)]} \tag{67}$$

By Corollary 2.1.1, $\Omega_A \leq 2\Lambda_A$ and therefore $\Omega_A \leq 2\Lambda_A^M$ as well. Similarly, $\Omega_B \leq 2\Lambda_B$, and $\Omega_B \leq 2\Lambda_B^M$.

The denominator of Equation 67 can be one of four values. If the denominator is Λ_A, which implies $\Lambda_A \geq \Lambda_B$, then Equation 67 reduces to:

$$\frac{\hat{z}}{z^*} \leq \frac{\Omega_A + \Omega_B}{\Lambda_A} \leq \frac{\Omega_A}{\Lambda_A} + \frac{\Omega_B}{\Lambda_B} \leq 2 + 2 = 4 \tag{68}$$

Similarly, when the denominator is Λ_A^M which implies $\Lambda_A^M \geq \Lambda_B^M$, then Equation 67 reduces to:

$$\frac{\hat{z}}{z^*} \leq \frac{\Omega_A + \Omega_B}{\Lambda_A^M} \leq \frac{\Omega_A^M}{\Lambda_A^M} + \frac{\Omega_B^M}{\Lambda_B^M} \leq 2 + 2 = 4 \tag{69}$$

And when the denominator is Λ_B which implies $\Lambda_B \geq \Lambda_A$, then Equation 67 reduces to:

$$\frac{\hat{z}}{z^*} \leq \frac{\Omega_A + \Omega_B}{\Lambda_B} \leq \frac{\Omega_A}{\Lambda_A} + \frac{\Omega_B}{\Lambda_B} \leq 2 + 2 = 4 \tag{70}$$

And when the denominator is Λ_B^M which implies $\Lambda_B^M \geq \Lambda_A^M$, then Equation 67 reduces to:

$$\frac{\hat{z}}{z^*} \leq \frac{\Omega_A + \Omega_B}{\Lambda_B^M} \leq \frac{\Omega_A^M}{\Lambda_A^M} + \frac{\Omega_B^M}{\Lambda_B^M} \leq 2 + 2 = 4 \tag{71}$$

$\square$

3.5 The Multiple Lane Case

We now consider locating a fixed number of rest facilities on multiple connected lanes. On such networks, drivers may be dispatched inbound to some terminal on one lane, and then outbound to another terminal on a different lane. Driver management is thus more difficult, and the related rest facility location problems are also more difficult. In this section, we consider the location problem, and in the next chapter, we investigate the dispatching problem more thoroughly.

3.5.1 Allocation of Facilities to Lanes

Allocation of rest facilities to lanes is the problem of determining how many facilities to locate on each lane. In general, the optimal allocation of a fixed number of rest facilities across multiple lanes cannot be determined without considering the load set, but there are exceptions. Consider the following cases consisting of two distinct single directional lanes with no interaction between the lanes, i.e., Pittsburgh to Boston and San Diego to San Francisco:

1. *Two 9-hour lanes, one rest facility to locate* - The lane on which the rest facility is not located will require two rests per load with a resulting cycle time of 38 hours per load. The lane on which the rest facility is located will require one rest on the first load and two rests on subsequent loads, if the facility is located optimally. The resulting cycle times for this lane will be 28 hours for the first load and 38 hours for every subsequent load. Now consider a load set consisting of two loads with pickup times of 0 and 28. If the two loads are on the lane with the rest facility, they can be delivered by a single driver. If the two loads are on the lane without the rest facility, however, then the load set will require two drivers. Thus, in this case, an optimal rest configuration cannot be determined without considering the load set.

2. *Two 9-hour lanes, two rest facilities to locate* - For each of the lanes, the cycle time sequence with no rest areas is at best 38 hours for every load. With one rest facility, the best cycle time sequence is (28,38) with the 38 repeating for each subsequent load. With two rest facilities, the cycle time sequence will be (28,28,38) with the last 2 cycle times (28,38) repeating for subsequent loads. In each case, more rest facilities results in a preferred cycle time sequence. In this case, it is clear that the optimal rest allocation to lanes will depend on the load set.

3. *Two 14-hour lanes, two rest facilities to locate* - For each of these lanes, one rest facility is required. The allocation decision is therefore trivial.

4. *One 20-hour lane and one 10-hour lane, three rest facilities to locate* - For the 20-hour lane, one rest facility is required. With a single rest area, a 20-hour lane will require four rests per load. With two rest facilities, the first load requires only three rests with every subsequent load requiring four rests. Using three rest facilities results in the same cycle time sequence as two rest facilities. The 10-hour lane does not require any rest areas and the resulting cycle time sequence has two rests per cycle. Increasing the number of facilities to one results in one rest on the first load and two rests on

every subsequent load. Using two rest areas results in the same cycle time sequence as one rest area. In this situation, locating two rest facilities on the 20-hour lane and one rest facility on the 10-hour lane is the optimal allocation.

As clearly indicated by these cases, whenever the cycle time sequences in both lanes can be improved by adding rest facilies, the allocation choice cannot be determined without considering the load set.

3.5.1.1 Myopic Facility Allocation Algorithm

Algorithm 3 presents a myopic algorithm for allocating facilities to lanes when the optimal allocation depends on the load set. The worst case performance of this algorithm depends on the network structure.

Theorem 3.6. *In a completely disjoint network where drivers cannot service loads on more than one lane, Algorithm 3 provides a solution which is at worst 4 times the optimal number of drivers.*

Proof. By Theorem 3.5 the solution on any particular lane is at most 4 times optimal for that lane. Since the lanes are disjoint, the network solution is the sum of the drivers on each lane. Therefore, by the distributive property, the solution is at most 4 times the sum of the single lane optimal solutions which is the optimal network solution. □

An example of when this bound is tight could not be found. When drivers can be dispatched on multiple lanes, the optimal solution may be less than the sum of the individual lane solutions, and thus the worst-case performance of this algorithm may be larger. The degree to which it is larger depends on the network structure. Consider a star network with four spokes. The optimal solution could be as low as the highest single lane lower bound, whereas the worst solution could use up to four times as many drivers on that lane alone, with another factor of four for drivers to service the other lanes resulting in a worst case performance of 16 times optimal.

Algorithm 3: Greedy Load Dependent Rest Facility Lane Allocation Algorithm

Inputs:

T, Set of terminals.

$LANES$, the set of origin-destination terminal pairs.

L^{OD}, the set of load pickup times for loads on lane (O, D) for all $(O, D) \in LANES$.

k, the total number of rest facilities to allocate between the lanes.

δ_{OD}, travel time from terminal O to terminal D for all $(O, D) \in LANES$.

Table 18, optimal rest facility locations and resulting rest time sequences for single lane.

Output:

$n_{OD}*$, Number of rest facilities to allocate to lane (O, D) for all $(O, D) \in LANES$.

Main:

SET $m(OD) = \lfloor \delta_{OD}/T_D \rfloor$, the minimum number of rest facilities required on lane OD, for every $(O, D) \in LANES$.

CALCULATE $m = \sum_{OD \in LANES} m(OD)/2$, the minimum number of rest facilities required.

CALCULATE $v = k - m$, the number of facilities available to distribute.

FOR each lane (AB) and its reciprocal lane (BA):

FOR each possible number of rest facilities, $i = m(AB)$ to m:

CALCULATE $\Lambda_{OD}(i)$, the lower bound on the number of drivers needed using Algorithm 2, the myopic rest facility location algorithm for a bi-directional lane, to locate the i facilities.

SET $n_{OD} = m_{OD}$, the number of facilities assigned to lane OD.

Assign the v available facilities to lanes one at a time as follows:

LOOP FOR facility $i = 1$ to v:

FIND the lane $\hat{OD}$ such that $\max[\Lambda_{OD}(n_{OD}) - \Lambda_{OD}(n_{OD} + 1)]$

SET $n_{\hat{OD}} = n_{\hat{OD}}$

NEXT i

END LOOP

EXIT

3.5.2 Locating Facilities on the Lanes

Since drivers' remaining drive hours at the beginning of a leg will be determined by their previous leg which may have been on a different lane, locating rest facilities on each lane is a much harder problem and in general cannot be determined independently from the load set.

Some general observations can be made, however, to help guide the placement of facilities. All else being equal, locating a facility closer to the source terminal is preferable. When this facility is used for the last rest on the return leg of a trip, the driver will have more remaining drive hours and could possibly reach rest facilities on other lanes he could not reach otherwise. Second, drivers returning from loads on other lanes are more likely to be able to reach the rest facility without resting first if the facility is closer to A. The degree to which this is a factor depends on the locations of the rest facilities on the other lanes.

For multiple rest facilities on a lane, the problem is further complicated. Consider an 18-hour lane with two rest facilities. The possible combination of optimal locations are illustrated in Figure 15. In this example, The range of optimal combinations of d_1 and d_2 make it possible to use the lowest optimal values of both d_1 and d_2. However, there may exist cases when there is a trade-off between the two variables. In other words, minimizing d_1 in the optimal range may require using a larger d_2 and vice versa. In this case, the correct locations to use cannot be so easily specified.

Another consideration for multiple rest facilities on a single lane is that a configuration which is suboptimal from a single lane perspective may allow for a facility closer to the source terminal which could allow for immediate turnaround when returning from another lane. Thus, in some circumstances, for example, on a less densely loaded lane, the flexibility afforded by the additional flexibility in driver turnaround from other lanes may be more than enough to counteract the negative effects of the suboptimal single-lane configuration. These concepts are further complicated by the fact that these dependencies between lanes are mutual. The optimal locations on lane X may depend on the locations on lane Y which depend on the locations on lane X. Yet another complicating factor is that both terminals on a lane may be sources for multiple lanes. It should be obvious from these complicating factors that the optimal locations for the rest facilities in a network may depend on the load set.

In the absence of a load-independent methodology for locating facilities, a reasonable approach would be to use $OPT(k)$ locations if they exist, and if not, $1L2OPT$ locations, then $1L1OPT$ locations. In other words, utilize the Algorithm 2 locations as the starting point. In each of these cases, when alternate optimal configurations exist, use the configuration which minimizes the distance from the closest rest facility to the lane source terminal, or the terminal with heaviest outbound lane flow. Algorithm 4 formalizes this procedure.

Algorithm 4: Myopic Rest Facility Location Algorithm

Inputs:
k, the number of rest facilities to locate on the lane.
Output:
$R^A(k)$, the algorithm recommended rest facility configuration.
Main:
Determine the Algorithm 2 load dependent solution for the k facilities.
Identify alternate optimal locations using Algorithm 1.
SET $R^A(k)$ =the alternate optimal configuration with the closest facility to the source terminal.

Local search methods could be used to find improvements to these locations. For example, one search neighborhood could be shifting the locations of facilities on two lanes in order to improve the interactions between the two lanes. Exploration of suboptimal single-lane locations when they improve the interactions between lanes could also be explored. Further exploration of these improvements will be left for future study.

When using Algorithm 4 on all lanes in a network, the worst case performance of this algorithm will depend on the network structure. The same bounds and reasoning used in the rest facility allocation algorithm also apply to this algorithm.

In the next two sections we will explore the rest facility allocation and location decisions for two specific multi-lane network types: a simple star network and two linearly consecutive lanes.

3.5.3 Star Networks

A simple star network consists of a single source, designated terminal A, with $n > 1$ destinations, designated B_i for $i = 1, 2, ..., n$. The length of each lane is denoted δ_{Ai} for $i = 1, 2, ..., n$. Each lane has a set of rest facilities, designated R_i for $i = 1, 2, ..., n$. The distance of the $j^t h$ facility on lane i from terminal A is designated d_j^i. The set of rest facility locations on lane i are designated $d(R_i)$. The set of all rest facility locations is designated $d(R)$ as before.

In general, lanes with very few loads should not receive any additional facilities beyond the minimum required for feasibility, whereas lanes with similar load characteristics should receive the same number of facilities. Algorithm 3 generates solutions with these features. The following is a basic procedure for locating k rest facilities on the lanes:

1. Start by allocating the k facilities to the lanes using Algorithm 3.

2. Locate facilities on each lane using Algorithm 4.

As discussed previously, when locating facilities on a lane when multiple optimal locations exist, using the location closest to A has some advantages. First, when this facility is used for the last rest on the return leg of a trip, the driver will have more remaining drive hours and could possibly reach rest facilities on other lanes he could not reach otherwise. Second, drivers returning from loads on other lanes are more likely to be able to reach the rest facility without resting first if the facility is closer to A.

3.5.4 Two Linear Consecutive Lanes

The two linear consecutive lanes network consists of three terminals, A, B, and C. Terminal B is on the path between A and C. Possible load origin-destination pairs are $A-B$, $A-C$, and $B-C$. The lane lengths are given by δ_{AB} for the distance between terminals A and B, δ_{BC} for the distance between B and C, and $\delta_{AC} = \delta_{AB} + \delta_{BC}$ for the distance between terminals A and C. The location of the j^{th} rest facility is measured in driving time from A, and is denoted d_j. The set of all rest facility locations is designated $d(R)$ as before.

There are several complicating factors in this scenario. First, for the A to C lane, terminal B is equivalent to an available rest facility fixed at position δ_{AB}. Second, when positioning facilities on either the A to B lane or the B to C lane, the effect on the A to C lane must be considered and vice versa. Third, drivers delivering loads from A to B do not necessarily return to A; they may be assigned a load from B to C. Fourth, drivers delivering loads to C, whether from A or B, may return to either A or B for their next load.

Optimal rest locations depend on the load set. If most of the loads are from A to B with only a few loads to C, the problem is very different from the problem when most of the loads are from A to C with only a few loads from A to B or B to C.

In the case of a lane with a predominance of the loads, it would seem intuitive to place only the minimum number of rest facilities for feasibility on the infrequently used lanes, and to use the MIP for the single lane location problem to optimally locate the remaining facilities on the predominant lane. This, however, ignores the interaction effects between the lanes.

It should be noted that when solving the MIP for locating k facilities on the lane AC, the MIP will have to be modified slightly by solving for $k+1$ facilities and fixing one of the facilities to be located at δ_{AB}.

In any case, when determining the optimal locations on a primary lane, the effects on the other lanes should be evaluated and small adjustments may be made if they can improve the resulting cycle time sequence on the other lanes without changing the cycle time sequence on the primary lane.

As an illustration, consider two consecutive 9-hour lanes AB and BC, with an 18-hour AC lane. Two rest facilities are to be located to optimize the AC lane. The MIP should be run for three rest facilities requiring one of the three rest facilities to be located at 9. The solution obtained is facilities located at 2 and 13 with a resulting cycle time sequence of (66,76) which repeats indefinitely. These locations, in turn, equate to locations of $d = 2$ for the AB 9-hour lane and $d = 4$ for the BC 9-hour lane. To keep the 18-hour lane cycle sequence the same, the rest area at 13 could actually be anywhere on the interval 12.5 to 13, and the rest area at 2 could be anywhere on the interval 1.5 to 2, with the additional constraint that $d_2 - d_1$ is no more than 11 hours. From Figure 11, the optimal location for a single facility on a 9-hour lane is anywhere on the interval 0 to 2. The rest facility on the AB lane meets this condition, but the rest facility on the BC lane does not and cannot be adjusted to meet this condition. The resulting locations for this scenario should therefore be 1.5 and 12.5.

Algorithm 5 is a greedy algorithm to sequentially allocate k facilities on the lanes for a specific load set using multi-cycle lower bounds.

Algorithm 5: Greedy k-Facility Allocation Algorithm for Linearly Consecutive Lanes

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Inputs:

k, the number of rest facilities to locate on the lane.

δ_{AB}, δ_{BC}, and δ_{AC}, the lane lengths.

L, the set of loads to be dispatched.

Output:

$R^A(k)$, the algorithm recommended rest facility configuration.

Main:

1. First allocate a minimum number of facilities to lanes AB and BC to ensure that drivers can feasibly move from origin to destination along these lanes; note then that lane AC is also feasible. Define the primary lane to be the one with the most loads to deliver. Locate these facilities to generate the best possible cycle time sequence on the primary lane, and then adjust locations as necessary within the range of alternative optimal locations in order to improve the cycle time sequence an any other lane affected by this facility. This is the baseline configuration.

2. Determine the cycle time sequences on each of the lanes, AB, BC, and AC for this baseline configuration.

3. Under the baseline configuration, determine multi-cycle lower bounds on the number of required drivers separately for the three lanes AB, BC, and AC.

4. Sequentially allocate the remaining rest facilities as follows:

   a. For each possible lane to which the facility could be added, optimally locate the facilities on that lane by resolving the MIP and making adjustments as necessary within the range of alternate optimal locations in order to improve the cycle time sequence an any other lane affected by this facility.

   b. Determine new multi-cycle lower bounds on each of the AB, BC and AC lanes for each of the three possible lane allocations.

   c. Assign the facility to the lane which produces the lowest sum of the multi-cycle lower bounds.

For example, if the adding the next facility to the AB lane results in multi-cycle lower bounds of 13, 14, and 12 on the AB, BC and AC lanes, respectively, and adding the facility to the BC lane results in multi-cycle lower bounds of 14, 11, and 10, and adding the facility to the AC lane results in lower bounds of 14, 13, and 10, then the facility should be added to the BC lane.

Rather than specifying worst case bounds on the performance of this heuristic, empirical test results will be reported in the next chapter.

3.6 Contributions

We now summarize the primary contributions of the research reported in this chapter.

- For a single lane with no backhauls:
  - Developed a mixed-integer programming approach for locating rest facilities to minimize the number of drivers needed to deliver a set of loads which takes into consideration hours of service requirements.
  - Constructed a table of optimal rest facility locations for up to four rest facilities for lane lengths from 5.5 to 22 hours.
  - Developed a procedure to identify sets of alternative optimal rest facility locations.
  - Developed for a single rest facility a graph of all optimal rest facility locations for lane lengths from 5.5 to 22 hours.
  - Showed that allowing empty trucks to rest anywhere does not provide any reduction in the number of drivers required for most lane lengths from 5.5 to 22 hours using up to three rest facilities.
- For a single lane with backhauls:
  - Proved two theorems regarding when optimum rest facility locations can be determined without considering the load set.
  - For a single rest facility, built a graph of all optimal rest facility locations for lane lengths from 5.5 to 22 hours.
  - Determined several sample optimal rest facility configurations for some lane lengths with up to four rest facilities.

- Presented a heuristic algorithm for locating facilities on the lane based on the load set and proved a bound on the worst case performance of this algorithm.

- For multiple lanes:
  - Demonstrated why optimal rest facility locations depend upon the load set.
  - Presented heuristic algorithms for allocating a designated number of facilities to lanes and for locating those facilities on the lanes for both a star network and for two consecutive lanes.

CHAPTER IV

THE MULTI-LANE DRIVER DISPATCHING PROBLEM WITH FIXED REST AREAS

4.1 Complicating Factors

In the multi-lane driver dispatching problem, there are several complicating factors not present in the single lane problem with no backhauls:

- Minimum driver solutions may require drivers to drive empty to the pickup location if no drivers are already available.
- The trade off between number of drivers and deadhead miles may not be clearly defined.
- Driver state sequences are more complex. Drivers have different state sequences depending upon the next lane upon which they will be dispatched.

To handle the final point, we will use a set of *lane states* defined uniquely determines the furthest rest area reachable by a driver in this state. Recall that the definition of a lane includes a source, a destination, and the lane length. Therefore the lane from A to B and the lane from B to A are considered to be distinctly separate lanes. A driver's remaining drive hours uniquely determines this state; it is possible, therefore, to specify a lower bound (LB) and an upper bound (UB) on the number of remaining drive hours a driver picking up a load on this lane must have at the pickup location in order to be in this lane state. The lane state determines how long it will take a driver to deliver a load, and how many hours the driver will drive on the last driving day for delivering this load. Thus, the lane state also uniquely determines the driver's remaining drive hours at the destination terminal.

A driver at a particular terminal will be in a different lane state for each possible subsequent lane. For example, a driver with eight remaining drive hours at terminal A may only be able to reach a rest area 6 hours away on the lane to terminal B, and a rest area

7 hours away on the lane to terminal C. All possible remaining drive hours for drivers at a particular terminal can be partitioned into ranges to define *terminal states*, which are essentially groups of lane states for each possible outbound lane. For any given terminal state, the lane state on each lane originating from that terminal can be determined. The following example illustrates the relationship between lane states and terminal states.

Example. Consider a terminal which is the origin for three 18-hour lanes where $\tau_D = 11$. Lane B_1 has rest facilities at locations 4 and 7. Lane B_2 has three rest facilities at 3, 11 and 14. The third lane, B_3, has a single rest facility at 7. The terminal states and associated lane states would be defined as follows for remaining drive hours, h_d:

- Terminal state 1 - Fully rested state ($h_d = 11$)
  - B_1 Furthest rest area: 7
  - B_2 Furthest rest area: 11
  - B_3 Furthest rest area: 7
- Terminal state 2 ($7 \leq h_d < 11$)
  - B_1 Furthest rest area: 7
  - B_2 Furthest rest area: 3
  - B_3 Furthest rest area: 7
- Terminal state 3 ($4 \leq h_d < 7$)
  - B_1 Furthest rest area: 4
  - B_2 Furthest rest area: 3
  - B_3 Furthest rest area: Not feasible
- Terminal state 4 ($3 \leq h_d < 4$)
  - B_1 Furthest rest area: Not feasible
  - B_2 Furthest rest area: 3
  - B_3 Furthest rest area: Not feasible

- Terminal state 5 ($0 \leq h_d < 3$)
  - B_1 Furthest rest area: Not feasible
  - B_2 Furthest rest area: Not feasible
  - B_3 Furthest rest area: Not feasible

Not feasible means a driver in this terminal state does not have sufficient drive hours to reach the first facility on that lane, and hence cannot deliver a load on that lane without resting first. In this example the combined driver lane state space was combined into five possible terminal states. Terminal states will be useful in the network flow formulations used later in this chapter.

4.2 Bounds

The following proposition establishes a simple upper bound on the minimum number of drivers needed for multi-lane networks.

Proposition 4.1. *An upper bound on the number of drivers needed for a given configuration of lanes and rest facilities for a given load set can be determined by optimally solving the single lane minimum driver dispatching problem for each lane independently, and then taking the sum of the single lane optimal number of drivers.*

Proof. The single lane optimal number of drivers on each lane is a feasible solution to the network problem, so more drivers are not needed.

The following instance demonstrates when this bound is tight. Consider k lanes, each with a single load to be picked up at time τ. The single lane optimal number of drives is 1 for each of the k lanes, so the network upper bound is k drivers. In this case, each of the k loads must be picked up at the same time, so all k drivers are needed and the bound is tight. □

The following proposition establishing a simple lower bound is clear and is presented without proof.

Proposition 4.2. *A lower bound on the number of drivers needed for a given configuration of lanes and rest facilities for a given load set can be determined by solving the single lane minimum driver dispatching problem for each lane independently, and then taking the largest single lane optimal number of drivers as a lower bound for the network.*

Since this lower bound involves solving a problem which can be computationally difficult, a slightly weaker, but polynomially computable bound can be obtained by taking the highest of the single lane multi-cycle bounds for each of the lanes. This weaker bound was used in analyzing the performance of the rest facility location and allocation algorithms presented in the previous chapter.

4.3 Determining Minimum Driver Solutions

The network flow methodology presented in Chapter 2 for single lane problems is now extended to handle multi-lane problems. The specific types of networks to be analyzed are the bidirectional lane, the simple star network and the two linearly consecutive lanes as illustrated in Figure 1.

4.4 The Bidirectional Lane

4.4.1 Problem Definition

A known set of loads, L, must be delivered from location A to location B or from location B to A. The driving time between locations A and B is δ_{AB} hours in either direction. Drivers can drive a maximum of τ_D hours before they must rest a minimum of τ_R hours. Each load ℓ has a designated dispatch time, t_ℓ, and pickup location, $a_\ell \in \{A, B\}$. Driver rest may only occur at locations A, B, and designated rest areas, $r \in R$, located d_r hours from location A. The minimum driver-to-load assignment problem is then to determine the minimum number of drivers required to serve all loads, and the assignment of this set of drivers to loads.

4.4.2 Bounds

An upper bound on the number of drivers required can be easily obtained by treating the sets of loads originating at A, denoted $L_A \subseteq L$, and the set of loads originating at B, denoted $L_B \subseteq L$, as independent problems and solving the minimum driver dispatching problem to determine the number of drivers required to deliver that subset assuming each driver's return leg will be empty. The sum of the two results would give an upper bound on the total number of drivers required; we now present these ideas formally.

Definition 4.1. *The **minimum number of drivers** needed to deliver a set of loads from terminal i to terminal j for given R is denoted z^*_{ij}.*

The following two propositions are trivial and presented without proof.

Proposition 4.3. *For a given rest area configuration, R, an upper bound on the number of drivers required to deliver loads $L_A \subseteq L$ from A to B, and loads $L_B \subseteq L$ from B to A, denoted $\Omega(\Leftrightarrow)$ is given by the following expression:*

$$z^* \leq z^*_{AB} + z^*_{BA} = \Omega(\Leftrightarrow) \tag{72}$$

Proposition 4.4. *For a given rest area configuration, R, a lower bound on the number of drivers required to deliver loads $L_A \subseteq L$ from A to B, and loads $L_B \subseteq L$ from B to A, denoted $\Lambda(\Leftrightarrow)$ is given by the following expression:*

$$z^* \geq \max\left(z^*_{AB}, z^*_{BA}\right) = \Lambda(\Leftrightarrow) \tag{73}$$

Clearly,

$$\left(\frac{\Omega(\Leftrightarrow)}{\Lambda(\Leftrightarrow)}\right) \leq 2 \tag{74}$$

4.4.3 Improved Bounds

4.4.3.1 The Occupation Bound

It may be possible to obtain a better lower bound on the number of drivers needed by determining the largest number of loads simultaneously in motion using the minimum lane transit time for the pickup location of the load. Each of these loads will require a different driver and thus represents a lower bound on the minimum number of drivers required. This bound easily extends to multiple lanes. The occupation bound will be denoted Λ_{OCC}.

4.4.3.2 Bidirectional Interactive Cycle Bound

Another bound introduced by Karacik [19] can be obtained by looking at the interactions of the loads from the two terminals. Specifically, count the number of loads originating from a terminal in the minimum round trip cycle time. Each of these loads will require a different driver. Additionally, all loads arriving from the other terminal during this interval will require unique drivers as well, except that these drivers may be dispatched on loads departing after their arrival. The bound is therefore the number of departing loads in the interval plus the number of arriving loads in the interval minus the number of drivers arriving in the interval who can be matched with subsequent departing loads in the interval. This bound will be called the Karacik bound and denoted Λ_K.

4.4.4 Network Flow Formulation with Bundling Constraints

The network flow formulation is similar to the one developed for the single lane with no backhauls problem with the following exceptions:

- Different driver states exist for each terminal, which are denoted terminal states.
- There are additional arc types. After delivering a load, a driver may either stay at the terminal where the load is delivered and be assigned another load there, or rest before being assigned another load there, or return and be assigned another load at the source, or return and rest before being assigned another load at the source.
- Drivers may be initially assigned loads at either A or B, so an unassigned arc is needed from the source node to the fully rested state for the first load originating at each terminal.
- The objective function is to minimize the sum of the flow on the unassigned arcs from the source to the fully rested state of the first load for each terminal.
- Unassigned arcs (U-arcs) flow from a given state for a given load to the same state for the next load at the same terminal.

- Assigned arcs (A-arcs) flow from a given state for a given load to the next state in the round trip state sequence for the first load after the cycle time at the same terminal.

- Assigned with rest arcs (AR-arcs) flow from a given state for a given load to the fully rested state at the first load after the round trip cycle time plus whatever time is needed to be fully rested at the same terminal.

- Assigned to other terminal arcs (A2-arcs) flow from a given state for a given load to the appropriate subsequent state for the load after the appropriate leg time at the other terminal. The leg time is defined by the terminal state in which the driver is dispatched.

- Assigned to other terminal with rest arcs (AR2-arcs) flow from a given state for a given load to the fully rested state for the load after the leg time and whatever time is required to become fully rested at the other terminal.

For the same number of loads and rest facilities, the network is larger than the single directional case because there are five outgoing arcs for each dispatch node instead of three, and there are more state nodes for each load. However, the problem is still practically solvable reasonably sized problems.

4.4.5 Computational Study

The network model was run on an 18-hour lane using 20 different load sets, 10 with exponentially distributed IAT's and 10 with uniformly distributed IAT's. In half of the load sets, loads were evenly split between the two terminals. In the other half of the load sets, the ratio of the number of loads originating from terminal A to the number of loads from terminal B was two. Each load set consisted of 500 loads. A summary of the load sets analyzed is presented in Table 22.

The lane was analyzed using one to five rest facilities. Algorithm 2 was used to locate the facilities. Specifically, if an $OPT(k)$ configuration for the k rest facilities existed, it was selected. Otherwise, if a $1L2OPT$ configuration existed either it or its mirror image

Table 22: Parameters of Load Sets Used in the Single Lane With Backhauls Computational Study

Load Sets	IAT Distribution	Mean IAT (min) Loads at A/B	IAT Range Loads at A/B	Number of Loads at A/B
2dE1-2dE5	Exponential	500/500	NA	250/250
2dU1-2dU5	Uniform	500/500	$100 - 900/100 - 900$	250/250
2dE6-2dE10	Exponential	500/1000	NA	333/167
2dU6-2dU10	Uniform	500/1000	$100 - 900/200 - 1800$	333/167

configuration was selected by Algorithm 2. In this case, results using both of these configurations are analyzed. The two configurations are differentiated in the results tables by labeling the one in which loads originating at A have the preferred leg time sequence as $1L2OPT(A, k)$, and the one in which loads originating at B have the preferred leg time sequence as $1L2OPT(B, k)$. The configuration selected by Algorithm 2 is bolded to aid in evaluating how well the algorithm performed. If neither an $OPT(k)$ nor a $1L2OPT$ configuration existed, then either the $1L1OPT(A, k)$ or its mirror image, the $1L1OPT(B, k)$ configuration was selected by Algorithm 2. Results using both of these configurations are analyzed, and the configuration selected by Algorithm 2 is bolded to aid in the evaluation of the algorithm's performance.

4.4.5.1 Results

The results of the computational study are provided in Tables 23 through 26. In those cases where no $OPT(k)$ configuration exists and one of the configurations was preferred by Algorithm 2, that configuration is in bold type. When one configuration resulted in fewer drivers, the drivers needed for that configuration is in bold type.

Some key observations:

- The minimum driver dispatching problem could be solved for most of these instances in under 10 seconds and all could be solved in under two minutes with up to five rest facilities.
- In the 21 scenarios in which no $OPT(k)$ configurations exist and there was a difference in the number of drivers required between optimizing for A or optimizing for B,

Table 23: Bounds, Optimal Number of Drivers, and Run Times by Load Set and Rest Configuration for a Single Lane With Backhauls with Exponentially Distributed IAT's with Equal Number of Loads at A and B

Load Set	Rest Configuration	Drivers Needed	$\Lambda(\Leftrightarrow)$	Run Time (sec)
E1	OPT(1)	21	21	1
E2	OPT(1)	26	21	1
E3	OPT(1)	19	18	1
E4	OPT(1)	20	20	1
E5	OPT(1)	23	20	1
E1	**1L1OPT(A,2)**	**20**	20	1
E2	1L1OPT(A,2)	24	21	1
E3	**1L1OPT(A,2)**	**17**	17	1
E4	**1L1OPT(A,2)**	**19**	19	1
E5	1L1OPT(A,2)	21	20	1
E1	1L1OPT(B,2)	21	21	1
E2	**1L1OPT(B,2)**	**23**	20	1
E3	1L1OPT(B,2)	18	18	2
E4	1L1OPT(B,2)	20	20	1
E5	**1L1OPT(B,2)**	21	18	1
E1	1L2OPT(A,3)	20	20	1
E2	1L2OPT(A,3)	24	20	1
E3	1L2OPT(A,3)	17	16	7
E4	1L2OPT(A,3)	19	19	2
E5	1L2OPT(A,3)	21	18	1
E1	1L2OPT(B,3)	20	20	1
E2	1L2OPT(B,3)	**23**	20	2
E3	1L2OPT(B,3)	17	16	2
E4	1L2OPT(B,3)	19	19	2
E5	1L2OPT(B,3)	21	18	2
E1	OPT(4)	20	20	4
E2	OPT(4)	21	20	20
E3	OPT(4)	16	16	12
E4	OPT(4)	19	19	44
E5	OPT(4)	19	17	8
E1	1L1OPT(A,5)	20	20	9
E2	1L1OPT(A,5)	21	20	46
E3	1L1OPT(A,5)	16	16	68
E4	1L1OPT(A,5)	19	19	94
E5	1L1OPT(A,5)	19	17	51
E1	1L1OPT(B,5)	20	20	10
E2	1L1OPT(B,5)	21	20	50
E3	1L1OPT(B,5)	16	16	19
E4	1L1OPT(B,5)	19	19	77
E5	1L1OPT(B,5)	19	17	58

BOLD rest configuration is the configuration selected by Algorithm 2.
BOLD drivers needed is strictly lower than the alternate configuration.

Table 24: Bounds, Optimal Number of Drivers, and Run Times by Load Set and Rest Configuration for a Single Lane With Backhauls with Uniformly Distributed IAT's with Equal Number of Loads at A and B

Load Set	Rest Configuration	Drivers Needed	$\Lambda(\Leftrightarrow)$	Run Time (sec)
U1	OPT(1)	15	13	1
U2	OPT(1)	16	16	1
U3	OPT(1)	14	13	1
U4	OPT(1)	15	13	1
U5	OPT(1)	15	14	1
U1	1L1OPT(A,2)	13	13	2
U2	**1L1OPT(A,2)**	**15**	15	1
U3	**1L1OPT(A,2)**	13	12	1
U4	1L1OPT(A,2)	14	13	1
U5	**1L1OPT(A,2)**	**13**	13	2
U1	1L1OPT(B,2)	13	13	1
U2	1L1OPT(B,2)	16	16	1
U3	1L1OPT(B,2)	13	13	2
U4	1L1OPT(B,2)	14	13	7
U5	1L1OPT(B,2)	14	14	2
U1	1L2OPT(A,3)	13	12	26
U2	1L2OPT(A,3)	15	15	3
U3	1L2OPT(A,3)	13	11	18
U4	1L2OPT(A,3)	14	12	5
U5	1L2OPT(A,3)	13	13	4
U1	1L2OPT(B,3)	13	12	4
U2	1L2OPT(B,3)	15	15	5
U3	1L2OPT(B,3)	13	11	23
U4	1L2OPT(B,3)	14	12	19
U5	1L2OPT(B,3)	13	13	5
U1	OPT(4)	12	12	53
U2	OPT(4)	15	15	88
U3	OPT(4)	12	11	51
U4	OPT(4)	12	12	59
U5	OPT(4)	13	13	109
U1	1L1OPT(A,5)	12	12	92
U2	1L1OPT(A,5)	15	15	77
U3	1L1OPT(A,5)	12	11	94
U4	1L1OPT(A,5)	12	12	111
U5	1L1OPT(A,5)	13	13	81
U1	1L1OPT(B,5)	12	12	96
U2	1L1OPT(B,5)	15	15	85
U3	1L1OPT(B,5)	12	11	96
U4	1L1OPT(B,5)	12	12	118
U5	1L1OPT(B,5)	13	13	92

BOLD rest configuration is the configuration selected by Algorithm 2.
BOLD drivers needed is strictly lower than the alternate configuration.

Table 25: Bounds, Optimal Number of Drivers, and Run Times by Load Set and Rest Configuration for a Single Lane With Backhauls with Exponentially Distributed IAT's with Twice as Many Loads at A than B

Load Set	Rest Configuration	Drivers Needed	$\Lambda(\Leftrightarrow)$	Run Time (sec)
E6	OPT(1)	21	20	1
E7	OPT(1)	18	18	1
E8	OPT(1)	22	22	1
E9	OPT(1)	18	17	1
E10	OPT(1)	19	17	1
E6	**1L1OPT(A,2)**	**18**	18	1
E7	**1L1OPT(A,2)**	**17**	16	1
E8	**1L1OPT(A,2)**	**21**	20	1
E9	**1L1OPT(A,2)**	17	16	1
E10	**1L1OPT(A,2)**	**16**	15	1
E6	1L1OPT(B,2)	20	20	1
E7	1L1OPT(B,2)	18	18	1
E8	1L1OPT(B,2)	22	22	1
E9	1L1OPT(B,2)	17	17	1
E10	1L1OPT(B,2)	17	17	1
E6	1L2OPT(A,3)	**18**	18	2
E7	1L2OPT(A,3)	17	16	2
E8	1L2OPT(A,3)	21	20	2
E9	1L2OPT(A,3)	17	16	2
E10	1L2OPT(A,3)	**16**	15	2
E6	1L2OPT(B,3)	19	18	2
E7	1L2OPT(B,3)	**16**	16	2
E8	1L2OPT(B,3)	21	20	2
E9	1L2OPT(B,3)	**16**	16	2
E10	1L2OPT(B,3)	17	15	1
E6	OPT(4)	18	18	17
E7	OPT(4)	16	16	24
E8	OPT(4)	20	20	3
E9	OPT(4)	16	16	20
E10	OPT(4)	15	15	7
E6	1L1OPT(A,5)	18	18	6
E7	1L1OPT(A,5)	16	16	16
E8	1L1OPT(A,5)	20	20	6
E9	1L1OPT(A,5)	16	16	16
E10	1L1OPT(A,5)	15	15	66
E6	1L1OPT(B,5)	18	18	7
E7	1L1OPT(B,5)	16	16	43
E8	1L1OPT(B,5)	20	20	5
E9	1L1OPT(B,5)	16	16	8
E10	1L1OPT(B,5)	15	15	33

BOLD rest configuration is the configuration selected by Algorithm 2.
BOLD drivers needed is strictly lower than the alternate configuration.

Table 26: Bounds, Optimal Number of Drivers, and Run Times by Load Set and Rest Configuration for a Single Lane With Backhauls with Uniformly Distributed IAT's with Twice as Many Loads at A than B

Load Set	Rest Configuration	Drivers Needed	$\Lambda(\Leftrightarrow)$	Run Time (sec)
U6	OPT(1)	13	13	1
U7	OPT(1)	13	13	1
U8	OPT(1)	13	13	1
U9	OPT(1)	15	15	1
U10	OPT(1)	14	14	1
U6	**1L1OPT(A,2)**	**12**	12	2
U7	**1L1OPT(A,2)**	**12**	12	2
U8	**1L1OPT(A,2)**	**12**	12	6
U9	**1L1OPT(A,2)**	**13**	13	5
U10	**1L1OPT(A,2)**	**13**	12	2
U6	1L1OPT(B,2)	13	13	1
U7	1L1OPT(B,2)	13	13	2
U8	1L1OPT(B,2)	13	13	1
U9	1L1OPT(B,2)	15	15	1
U10	1L1OPT(B,2)	14	14	1
U6	1L2OPT(A,3)	12	12	15
U7	1L2OPT(A,3)	12	12	3
U8	1L2OPT(A,3)	**12**	12	2
U9	1L2OPT(A,3)	13	13	3
U10	1L2OPT(A,3)	13	12	3
U6	1L2OPT(B,3)	12	12	4
U7	1L2OPT(B,3)	12	12	3
U8	1L2OPT(B,3)	13	12	11
U9	1L2OPT(B,3)	13	13	3
U10	1L2OPT(B,3)	13	12	2
U6	OPT(4)	12	12	73
U7	OPT(4)	12	12	68
U8	OPT(4)	12	12	69
U9	OPT(4)	13	13	42
U10	OPT(4)	13	12	12
U6	1L1OPT(A,5)	12	12	64
U7	1L1OPT(A,5)	12	12	24
U8	1L1OPT(A,5)	12	12	50
U9	1L1OPT(A,5)	13	13	117
U10	1L1OPT(A,5)	13	12	54
U6	1L1OPT(B,5)	12	12	96
U7	1L1OPT(B,5)	12	12	89
U8	1L1OPT(B,5)	12	12	98
U9	1L1OPT(B,5)	13	13	74
U10	1L1OPT(B,5)	13	12	21

BOLD rest configuration is the configuration selected by Algorithm 2.
BOLD drivers needed is strictly lower than the alternate configuration.

Algorithm 2 chose the optimal configuration in 15 of those 21 scenarios. In the other 6 scenarios, Algorithm 2 was indifferent.

- In the scenarios in which Algorithm 2 preferred one configuration over another, there were no cases in which the non-preferred configuration resulted in fewer required drivers.

- Adding backhauls resulted in an increase of at most five drivers over the highest single direction minimum number of drivers. This represents at most a 24% increase. For the load sets with equal numbers of loads originating at A and B, there was no increase in the number of drivers for 45 of the 80 load set rest area combinations. For the load sets with twice as many loads originating at A, there was no increase in the number of drivers for 59 of the 80 load set rest area combinations.

- In those scenarios in which no $OPT(k)$ configurations exist, the difference between the number of drivers needed when the locations are optimized for A and when the locations are optimized for B is no more than two drivers, representing at most a 15% increase.

- In 39 of the 60 scenarios where no $OPT(k)$ configurations exist, the difference in the number of drivers needed between the configuration optimized for A and the configuration optimized for B was zero.

- In only 2 of the 30 scenarios where no $OPT(k)$ configurations exist and more loads originate from A did the configuration optimized for B produce a better solution.

In summary, when more loads originate from one of the terminals and no $OPT(k)$ rest configuration exists, then optimizing the rest facilities for the lane with the largest number of loads will require fewer drivers for most load sets. Also, when Algorithm 2 prefers a configuration over another, it should be used. Furthermore, the network flow model with bundling constraints is tractable for a moderate number of rest facilities. For this single lane case, Algorithm 2 was not really needed in that each possible configuration could be solved in a reasonable amount of time and the configuration producing the minimum driver solution could be selected. But for larger networks in which single lanes with backhauls are the building blocks, solving for the minimum driver solution to optimality for multiple configurations on each individual lane is impractical. In those cases, Algorithm 2, which only needs to calculate four distinct multi-cycle lower bounds, is a much faster and more practical solution method, and as this study demonstrated, an effective one as well.

4.5 The Star Network

4.5.1 Problem Definition

A known set of loads, L, must be delivered from location A to a designated location B_i. The driving time between locations A and a given destination terminal B_i is δ_{Ai} hours in either direction. Drivers can drive a maximum of τ_D hours before they must rest a minimum of τ_R hours. Each load ℓ has a designated dispatch time, t_ℓ, and destination location, $a_\ell \in \{B_1, B_2, ..., B_b\}$. Driver rest may only occur at locations A, B, and designated rest areas, $r \in R$, located d_r hours from location A on the lane to terminal B. The minimum driver-to-load assignment problem is then to determine the minimum number of drivers required to serve all loads, and the assignment of this set of drivers to loads.

4.5.2 Network Flow Formulation

The network flow formulation for a star network is very similar to the single lane with no backhauls network flow formulation, with the following differences:

- Lane and terminal states are again used, since furthest rest area states depend on the next outbound lane for dispatch.
- The duration of the A-arcs and the AR-arcs will depend on the destination terminal for the given load.
- The subsequent terminal state at A for the A-arcs will depend on the lane to which the load is assigned which in turn will determine the number of drive hours remaining.

The complexity of the problem depends on the number of distinct driver terminal states. For any number of identical lanes with identical rest facility configurations, the problem is no more complex than a single such lane with the resulting higher load density.

Table 27: Scenarios and Load Set Parameters used in the Star Network Computational Study

Scenario	Load Sets	9A/9B/18 Mean IAT (min)	9A/9B/18 Loads
1	5	500/500/500	250/250/250
2	5	500/500/1000	250/250/125
3	5	1000/1000/500	125/125/250
4	5	1000/500/500	125/250/250

4.5.3 Computational Study

For the star network, we are interested in investigating how the allocation of a fixed number of rest facilities to lanes may affect the number of drivers required to deliver a set of loads. In this computational study we will look at a star network consisting of three lanes: two 9-hour lanes designated lane 9A and 9B, and one 18-hour lane designated 18. The study will explore how two rest facilities should be allocated to the lanes. The 18-hour lane requires a minimum of one rest facility for feasibility, so the study will explore the effects of locating the extra facility on each of the three lanes. We will consider cases in which the lanes have similar load characteristics as well as cases in which loads are not evenly balanced between lanes. A summary of the four scenarios and associated load instances analyzed is presented in Table 27. All IAT's in this computational study are uniformly distributed within the mean ±80%.

Detailed results for each scenario are presented in Tables 28 through 31. The lane chosen to receive the extra facility by Algorithm 3 is marked with an asterisk. In those cases where Algorithm 3 is indifferent between choosing two or more lanes for the extra facility, no lane is highlighted.

For scenario 1, as indicated by Table 28:

- All instances could be solved in less than 4 seconds.
- Placing the extra rest facility on the 18-hour lane only produced the minimum number of drivers for load sets 3 and 5. For these load sets, however, the same number of drivers were needed regardless of which lane the rest facility was located.
- In this scenario, locating the facility on lane 9A required the minimum number of

Table 28: Optimal Number of Drivers and Run Times by Load Set and Rest Configuration for a 3-Lane (9A, 9B, 18) Star Network with Identical Load Distributions per Lane (Scenario 1)

Load Set	Lane with Extra RF	Drivers Needed	Run time (sec)
1	$9A$	22	2
1	$9B$	22	2
1	18	23	2
2	$9A$	24	2
2	$9B$	25	2
2	18	25	1
3	$9A$	24	4
3	$9B$	24	2
3	18	24	1
4	$9A*$	24	2
4	$9B$	26	2
4	18	26	1
5	$9A$	24	2
5	$9B$	24	1
5	18	24	1

$*=$ Lane chose by Algorithm 3 for the extra rest facility.

drivers for each load set. However, since the load set parameters for lane 9A and 9B were identical, in each case where locating the facility on 9A required fewer drivers than locating the facility on 9B, switching the load sets between the lanes would also switch the preferred lane for the facility. The optimal lane is therefore dependent on the load set.

- In this scenario, locating the facility on the wrong lane required no more than 8% more drivers, and resulted in at most a 2 driver increase.

For scenario 2, as indicated by Table 29:

- All instances could be solved in less than 6 seconds.
- Placing the extra rest facility on the 18-hour lane only produced the minimum for load set 2. For this load set, however, the same number of drivers were needed regardless of which lane the rest facility was located.
- The best lane to locate the rest facility depends on the load set. For load set 1, locating the facility on lane 9B resulted in fewer drivers, whereas for load sets 3 and

Table 29: Optimal Number of Drivers and Run Times by Load Set and Rest Configuration for a 3-Lane (9A, 9B, 18) Star Network with Fewer Loads on the 18-Hour Lane (Scenario 2)

Load Set	Lane with Extra RF	Drivers Needed	Run time (sec)
1	$9A$	19	1
1	$9B*$	18	2
1	18	19	1
2	$9A$	18	2
2	$9B*$	18	1
2	18	18	1
3	$9A$	20	1
3	$9B$	22	1
3	18	22	1
4	$9A*$	18	6
4	$9B$	19	2
4	18	20	1
5	$9A*$	18	2
5	$9B$	18	2
5	18	19	1

$*$ = Lane chose by Algorithm 3 for the extra rest facility.

4, locating the facility on lane 9A resulted in fewer drivers.

- In this scenario, locating the facility on the wrong lane required no greater than 11% more drivers, and resulted in at most a 2 driver increase.

For scenario 3, as indicated by Table 30:

- All instances could be solved in less than 1 second.
- Placing the extra rest facility on the 18-hour lane produced the minimum for all but load set 4. For this load set, locating the facility on lane 9A required fewer drivers.
- The best lane to locate the rest facility depends on the load set. For load set 4, locating the facility on lane 9A resulted in fewer drivers, whereas for load set 5, locating the facility on lane 18 required fewer drivers.
- In this scenario, locating the facility on the wrong lane required no greater than 6% more drivers, and resulted in at most a 1 driver increase.

For scenario 4, as indicated by Table 31:

Table 30: Optimal Number of Drivers and Run Times by Load Set and Rest Configuration for a 3-Lane (9A, 9B, 18) Star Network with Fewer Loads on Each 9-Hour Lane (Scenario 3)

Load Set	Lane with Extra RF	Drivers Needed	Run time (sec)
1	$9A$	19	1
1	$9B$	20	1
1	18	19	1
2	$9A$	18	1
2	$9B$	19	1
2	$18*$	18	1
3	$9A$	19	1
3	$9B$	19	1
3	$18*$	19	1
4	$9A$	18	1
4	$9B$	19	1
4	18	19	1
5	$9A$	20	1
5	$9B$	20	1
5	18	19	1

$*$ = Lane chose by Algorithm 3 for the extra rest facility.

Table 31: Optimal Number of Drivers and Run Times by Load Set and Rest Configuration for a 3-Lane (9A, 9B, 18) Star Network with Fewer Loads on the First 9-Hour Lane (Scenario 4)

Load Set	Lane with Extra RF	Drivers Needed	Run time (sec)
1	$9A$	22	1
1	$9B*$	21	1
1	18	22	1
2	$9A$	22	1
2	$9B*$	21	1
2	18	22	1
3	$9A$	23	1
3	$9B*$	22	1
3	18	22	1
4	$9A$	21	1
4	$9B$	22	2
4	18	22	1
5	$9A$	21	1
5	$9B*$	21	2
5	18	21	1

$*$ = Lane chose by Algorithm 3 for the extra rest facility.

- All instances could be solved in less than 2 second.

- Placing the extra rest facility on the 9-hour lane with fewer loads (9A) required fewer drivers for load set 4, but more drivers for load sets 1, 2 and 3. For load set 5, all locations required the same number of drivers. This illustrates that the optimal location depends on the load set.

- In this scenario, locating the facility on the wrong lane requiredno greater than 5% more drivers, and resulted in at most a 1 driver increase.

For each of the 11 load sets in which Algorithm 3 generated a clearly preferred lane on which to locate the rest facility, that location was optimal. There were nine load sets in which the algorithm was indifferent. Furthermore, the chosen lane was one of the lanes one would expect to choose based on load distributions between lanes. In scenario 1 when loads were evenly distributed between the lanes, only for one of the five load sets did the algorithm produce a preferred lane. In scenario 2 when the 18-hour lane had fewer loads, the algorithm preferred the location on 9A twice and on 9B twice. In scenario 3 when both 9-hour lanes had fewer loads, the algorithm chose the 18-hour lane to receive the extra facility twice and was indifferent for the other three load sets. In scenario 4 where the first 9-hour lane (9A) had fewer loads, the algorithm chose 9B to receive the extra facility for four of the load sets and was indifferent for the other load set. This result indicates that the lane length also plays a factor in choosing the lane to receive the facility. When a short and a long lane have the same number of loads, only the shorter lane was ever chosen to receive the extra rest facility. One possible explanation for this is that the rest facility on the shorter lane will provide more flexibility because the driver will return sooner. In other words, even though the longer lane has the same number of loads, the drivers will be servicing each load for a much longer period of time.

Overall, the dependence of the optimal lane to locate the facility on the load set was clearly illustrated in this computational study. However, the effects of locating the facility on the wrong lane required at most 11% more drivers. Furthermore, the computability of this problem using the network flow formulation with bundling constraints was illustrated for a small number of rest facilities on a reasonable number of lanes with an average run time of 1.4 seconds for the instances in this study. Furthermore, placing the rest facility on one of the lanes with more loads clearly produced better results than locating the facility on a lane with fewer loads and when the lanes with more loads are of different length, placing the facility on the shorter lane is more likely to require fewer drivers. The degree to which this difference in lane length plays a role as a function of the difference in lane length is left for future study.

4.6 Two Linearly Consecutive Lanes

4.6.1 Problem Definition

The two linearly consecutive lanes network consists of three terminals, A, B, and C. Terminal B is on the path between A and C. Loads $L_{AB} \subseteq L$ must be delivered from A to B, loads $L_{AC} \subseteq L$ must be delivered from A to C, and loads $L_{BC} \subseteq L$ must be delivered from B to C. The driving time between A and B is δ_{AB} in either direction. The driving time between B and C is δ_{BC} in either direction, and the driving time between A and C is $\delta_{AC} = \delta_{AB} + \delta_{BC}$ in either direction. Drivers can drive a maximum of τ_D hours before they must rest a minimum of τ_R hours. Each load ℓ has a designated dispatch time, t_ℓ, pickup location, $a_\ell \in \{A, B\}$, and destination location, $b_\ell \in \{B, C\}$. Driver rest may only occur at locations A, B, C and designated rest areas, $r \in R$, located d_r hours from location A. The minimum driver-to-load assignment problem is then to determine the minimum number of drivers required to serve all loads, and the assignment of this set of drivers to loads.

4.6.2 Network Flow Formulation

The network flow formulation for two linearly consecutive lanes combines some of the characteristics of both the star network and the single lane with backhauls network flow formulations. Like the single lane with backhauls, loads can originate from either A or B. Therefore, different states must be defined for each potential source terminal and A-arcs and AR-arcs must be defined for both possible subsequent load pickup terminals (A or B). Like the star network, loads originating at A may be dispatched to different terminals, so the state nodes will be associated with ranges of remaining drive hours with the durations

Table 32: Scenarios and Load Sets Used in the Linearly Consecutive Lanes Computational Study

Scenario	Load Sets	AB/BC/AC Mean IAT (min)	AB/BC/AC Loads
1	5	500/1000/1000	250/125/125
2	5	1000/500/1000	125/250/125
3	5	1000/1000/500	125/125/250
4	5	500/500/500	250/250/250

and termination nodes for the A-arcs and AR-arcs depending on the loads destination and the driver's initial remaining drive hours.

4.6.3 Computational Study

In this computational study we will study the placement of two rest facilities on two consecutive 9-hour lanes (and the resulting 18-hour lane). Four scenarios will be evaluated: when the AB lane is the predominant lane (Scenario 1), when the BC lane is the predominant lane (Scenario 2), when the AC lane is the predominant lane (Scenario 3), and when all lanes have the same number of loads (Scenario 4). Five load set instances for each scenario will be evaluated. A summary of the load set characteristics is presented in Table 32.

The performance of Algorithm 5 presented in the previous chapter for locating rest facilities on two consecutive lanes will be evaluated. For each of these scenarios, solutions for four different rest facility configurations will be calculated. In the first configuration, both facilities will be used to optimize the AB 9-hour lane cycle time sequence ($d_1 = 4$ and $d_2 = 7$). In the second, both facilities will be used to optimize the BC 9-hour lane cycle time sequence ($d_1 = 13$ and $d_2 = 16$). In the third, both rest facilities will be used to optimize the 18-hour lane cycle time sequence ($d_1 = 2$ and $d_2 = 13$). In the final configuration, one facility will be used to optimize the cycle time sequence in each of the two 9-hour lanes ($d_1 = 2$ and $d_2 = 11$). The detailed results for these scenarios are presented in Tables 33 through 36. The configuration chosen by the allocation algorithm is indicated with an asterisk.

The most interesting observation from this computational study was that the rest area configuration which used a single rest facility to optimize each of the two 9-hour lanes dominated all other configurations for every load set in every scenario. Also, the configuration

Table 33: Optimal Number of Drivers and Run Times by Load Set and Rest Configuration for 2 Consecutive 9-Hour Lanes with More Loads on the First 9-Hour Lane (Scenario 1 - AB Lane Predominant)

Load Set	Lane Optimized for RFs	Drivers Needed	Run time (sec)
1	AB	15	13
1	BC	16	1
1	AC	15	3
1	AB/BC^*	14	5
2	AB	17	16
2	BC	18	1
2	AC	17	3
2	AB/BC^*	16	4
3	AB	16	6
3	BC	18	1
3	AC	16	17
3	AB/BC^*	16	6
4	AB	14	19
4	BC	16	1
4	AC	14	17
4	AB/BC^*	14	25
5	AB	15	3
5	BC	16	1
5	AC	15	3
5	AB/BC^*	14	27

$*$ = Configuration chosen by Algorithm 5 .

Table 34: Optimal Number of Drivers and Run Times by Load Set and Rest Configuration for 2 Consecutive 9-Hour Lanes with More Loads on the Second 9-Hour Lane (Scenario 2 - BC Lane Predominant)

Load Set	Lane Optimized for RFs	Drivers Needed	Run time (sec)
1	AB	16	2
1	BC	16	1
1	AC	16	2
1	AB/BC^{*}	15	3
2	AB	16	2
2	BC	17	2
2	AC	16	4
2	AB/BC^{*}	16	5
3	AB	15	2
3	BC	17	1
3	AC	15	20
3	AB/BC^{*}	14	29
4	AB	15	2
4	BC	16	2
4	AC	15	3
4	AB/BC^{*}	14	3
5	AB	15	2
5	BC	14	1
5	AC	15	22
5	AB/BC^{*}	13	28

$*$ = Configuration chosen by Algorithm 5 .

Table 35: Optimal Number of Drivers and Run Times by Load Set and Rest Configuration for 2 Consecutive 9-Hour Lanes with More Loads on the Combined 18-Hour Lane (Scenario 3 - AC Lane Predominant)

Load Set	Lane Optimized for RFs	Drivers Needed	Run time (sec)
1	AB	18	20
1	BC	21	1
1	AC	18	22
1	AB/BC^*	17	3
2	AB	20	3
2	BC	21	1
2	AC	20	3
2	AB/BC^*	20	3
3	AB	19	3
3	BC	19	1
3	AC	19	4
3	AB/BC^*	18	2
4	AB	17	4
4	BC	19	2
4	AC	17	23
4	AB/BC^*	17	3
5	AB	17	6
5	BC	19	1
5	AC	17	3
5	AB/BC^*	16	36

$*$ = Configuration chosen by Algorithm 5 .

Table 36: Optimal Number of Drivers and Run Times by Load Set and Rest Configuration for 2 Consecutive 9-Hour Lanes with No Predominant Lane (Scenario 4)

Load Set	Lane Optimized for RFs	Drivers Needed	Run time (sec)
1	AB	24	53
1	BC	23	2
1	AC	24	7
1	AB/BC^*	22	18
2	AB	24	6
2	BC	24	2
2	AC	24	7
2	AB/BC^*	22	128
3	AB	23	4
3	BC	25	2
3	AC	23	64
3	AB/BC^*	23	79
4	AB	22	56
4	BC	24	2
4	AC	22	52
4	AB/BC^*	21	29
5	AB	22	20
5	BC	23	2
5	AC	22	33
5	AB/BC^*	21	55

$* =$ Configuration chosen by Algorithm 5 .

which used both facilities to optimize the BC lane required the most drivers for every load set in every scenario except for load set 5 in scenario 2. The largest difference between the number of drivers needed for a given load set due to different rest configurations was four drivers (or 24%). The largest difference between the number of drivers needed using a strategy of assigning all facilities to the predominant lane and the optimal was three drivers when the BC lane was predominant, one driver when the AC lane was predominant, and one driver when the AB lane was predominant.

For a strategy of assigning the facilities to the predominant lane to produce better results, larger differences in load densities between the load sets would be needed. Clearly, when all the loads are on the predominant lane, optimizing the rest facility configuration based on that lane will require the fewest drivers. An open question is then: what percentage of the total loads must be on other lanes for this strategy to fail, and what are the factors affecting that point? In other words, what is required for a lane to be considered predominant with respect to the rest facility location decision? This is an area worthy of further study.

The suggested procedure for allocating the facilities using Algorithm 5 performed very well. For each of the 20 load sets analyzed, the algorithm chose the configuration which required the fewest drivers. More specifically, it chose to use one facility to optimize the AB lane and one facility to optimize the BC lane. In looking closer at how the algorithm performed, for 18 of the 20 load sets, the first rest facility was placed on the AB lane and the second on the BC lane. For one load set the first facility was placed on the BC lane and the second on the AB lane. For the other load set, the algorithm was indifferent whether the first facility was on the AB or the BC lane, but for each of these possible choices, the second facility was placed on the other lane, resulting in the same configuration. In looking closer at the load sets and their multi-cycle lower bounds, the reason for these choices becomes clear. In each load set, placing a single facility on the AB lane improved both the AB lane bound and the AC lane bound as much as either of these bounds could be improved. In other words, adding a second or even a third facility to the AB lane would still result in the same multi-cycle lower bound for both the AB and the AC lanes. However, adding the facility to the AB lane had no effect on the BC lane. Alternatively, adding a single facility to the BC lane provided the absolute minimum lower bound on the number of drivers required on the BC lane, but added no benefit to either the AB or the AC lane. Therefore, only in the case when the BC lane was the predominant lane did the benefits from adding the first facility to the BC lane override the combined benefits to the AB and AC lanes of adding the first facility to the AB lane, and this occurred only in 1 of the 5 load sets in this scenario.

Different load sets could have produced different results since the cycle time sequences could still be improved by adding a second facility. For the load sets in this computational study, however, a single facility was sufficient to achieve the minimum possible multi-cycle lower bound on the number of drivers.

This study demonstrates the necessity to evaluate the possible rest facility configurations with expected load sets to determine the preferred rest facility configuration whenever multiple rest facilities are to be located on networks with multiple lanes.

4.7 Contributions

We now summarize the primary contributions of the research reported in this chapter.

- Developed a network flow formulation with bundling constraints and demonstrated it to be a viable method for solving problems with limited numbers of load source locations with a reasonable number of rest facilities on each lane. This was demonstrated on the following networks:
  - Bidirectional lane (single lane with backhauls).
  - Star network.
  - Two linearly consecutive lanes.
- Demonstrated that optimal rest facility locations are dependent on the expected load sets for most typical multi-lane scenarios.
- Demonstrated that the suggested algorithms for allocating and locating rest facilities between lanes perform very well empirically.

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